The stability of low temperature magnetic phases of YbIG in dependence on field and temperature is investigated using the model with six Yb sublattices and one saturated Fe sublattice. The conditions for existence of zero-frequency modes are deduced in the analytical form for magnetic fields lying in \{110\} planes. General results are applied to the case of magnetic field parallel to [111] direction and the existence of a metastability region is shown, where both the colinear as well as the canted phase may occur.

1. INTRODUCTION

The ground doublet splitting of Yb ions in iron garnet is strongly dependent on the direction of Fe ions sublattice magnetization (hereafter denoted by \( \mathbf{M} \)). Therefore, YbIG represents a ferrimagnet with large magnetic anisotropy [1]. In zero external magnetic field, eight \( \langle 111 \rangle \) directions are the easy directions of \( \mathbf{M} \). The external magnetic field \( H \) will deviate, generally, the possible equilibrium directions of \( \mathbf{M} \) from the \( \langle 111 \rangle \) directions and will make them energetically inequivalent. In dependence on \( H \) and temperature \( T \), various situations of \( \mathbf{M} \), \( H \) and Yb ions magnetization with respect to each other are realized. These various magnetic phases and the transitions between them were tested experimentally [2, 3, 4] and for the theoretical description of them various models were used [2, 5]. In Alben's paper [5], the regions of the different magnetic phases in \( (H - T) \) plane were delimited by a thorough numerical computation. The numerical procedure used there consisted in finding the lowest minima of Gibbs' free energy corresponding to the system of six rare-earth sublattices and one saturated iron sublattice. The states of the iron sublattice were described there by the direction angles of the saturated magnetization and the statistics of the rare earth sublattices was assumed to be determined by the commonly used mean-field spin Hamiltonian.

In the present paper, the existence of soft modes and the stability of magnetic phases in dependence on \( H \) and \( T \) is investigated. Within the frame-work of the mentioned six rare-earth sublattices model, the torque acting on \( \mathbf{M} \) for small deviations from equilibrium direction is determined. Then, the analytical conditions for the existence of zero frequency modes are deduced from the classical equations of motion of \( \mathbf{M} \) which determine the homogeneous mode frequency \( \omega \) as a function of \( T \) and \( H \). The curves \( \omega(T, H) = 0 \) define the instability boundaries of magnetic phases. As an example, the metastability region of colinear and non-colinear magnetic phase is deduced for the case \( H \parallel [111] \) and the transition of this region into the critical point is discussed.

2. THE EFFECTIVE FIELD

The model being dealt with supposes the Yb ions to be independent on each other and their ground state doublet splitting to be given in the mean field approximation

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by the following Hamiltonian [6]:

\[ \mathcal{H}_j = (\mu_B H g_j - MG_j) s_j \equiv F_j \sigma_j. \]

\( s_j \) represents the fictitious spin (in units \( h \)) belonging to ground doublet states of an Yb ion, \( F_j \) has the meaning of the effective field acting on \( s_j \). The index \( j = 1, 2, \ldots, 6 \) is related to six dodecahedral sites of the garnet structure. The tensors \( G_j, g_j \) are supposed to have identical principal axes \((x_j, y_j, z_j)\) and their components are usually drawn from spectroscopic measurements \([6, 7]\). The energy levels of the ground state doublet are thus equal to

\[ E_j^\pm = \pm \frac{1}{2} |F_j| \]

The relation (2) may be used not only for the static case but also for situations where the quantum-mechanical adiabatic approximation is applicable. This is true, e.g., for microwave frequencies used in ferromagnetic resonance \([8]\).

Owing to the fact that the interactions within the Fe-sublattice are much stronger than the interactions of Yb ions with Fe ions, the internal state of the Fe-sublattice is assumed to be constant — independent from changes in Yb sublattices. Accordingly, only variations of \( M \) leaving \( |M| \) constant are considered.

Our consideration will be based on the classical equation of motion for the Fe-sublattice magnetization. The total torque acting on \( M \) results from the interaction of Fe-spins with the external magnetic field, with the lattice and with the Yb ions. The direct interaction of Fe spins with the lattice — which gives rise to the magneto-crystalline anisotropy in YIG — will be neglected in the subsequent treatment since it is very small compared to the anisotropy interaction given by one-ion Hamiltonian \((1)\) \([9, 1]\). The effective field and the corresponding torque arising from the latter interaction is determined by the work done by the system of Yb ions on the Fe-sublattice when \( M \) is varied. Therefore, variations of the internal energy of Yb ions during a change of the magnetization direction has to be examined.

According to the considerations at the beginning of this section, the mean energy of the system of Yb ions is given at any time as follows:

\[ \bar{E} = - \sum_{j=1}^{6} \frac{1}{2} n_j \Delta_j. \]

Here, \( \Delta_j = E_j^+ - E_j^- \), and \( n_j \) means the instantaneous difference in populations of the doublet levels for ions in the \( j \)-th position. The internal energy of Yb ions, dependent on the arrangement of the magnetic moments only \([10]\), is obtained by subtracting the potential energy of Yb ions in the external magnetic field from (3). We shall use the internal energy per unit volume, i.e.,

\[ \mathcal{U} = \bar{E} + HM', \]

where \( M' \) represents the total magnetization of Yb ions explicitly given by Eq. (A3) of Appendix.