Coulomb-Assisted Cold Fusion

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When taking into account the energy-momentum exchange with a catalyzing lattice nucleus, the Coulomb barrier penetrability for the fusing nuclei becomes of order unity.

As is well known, the recently published experimental results appear to be incompatible with the extant theoretical attempts at explaining the observed reaction rates, which exceed the theoretical rates by a factor $10^{20} - 10^{40}$. The present paper is devoted to identifying and describing a mechanism which has the potential of bridging the gap between the experimental and the theoretical reaction rates. However, in this paper no other questions will be addressed, in particular, the question on which of the possible reactions are responsible for the observations.

Even stripping the process down to the simplest ingredients, the cold fusion process involves three nuclei, viz., the two fusing nuclei and the (catalyzing) nucleus sharing the liberated energy and momentum. Therefore an exact treatment of this stripped-down description would demand the solution of a full-fledged three-body problem. We discuss here this problem by means of perturbation treatment. As we show, the qualitative aspects of the reaction are fully contained in the results of such a treatment.

Precisely, we use here the following perturbation scheme. (i) We treat the fusing nuclei as a two-body system taking into account the mutual Coulomb repulsion. (ii) We account for the energy sharing with the third particle, viz., the catalyzing lattice nucleus—which is needed for overall energy-momentum conservation—by perturbation in first order. Thus, within our assumptions the fusion process is described by the two Feynman graphs, Fig. 1a and b. In the reaction according to Fig. 1a the fusion takes place first, while the energy-momentum sharing takes place after the fusion. In Fig. 1b the energy-momentum exchange takes place before the fusion, and the fusion reaches directly the ground state of

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Fig. 1. Feynman graphs for the radiationless fusion. $m_1$—catalyzing (lattice) nucleus; $m_2$, $m_3$—fusing nuclei; $\psi_0(t)$—momentum space “trapping” wave function; $F(q)$, $f(q)$, $g(q)$—electron scattering form factors; $\psi(p)$—momentum space fusion channel wave function. (a) Recoil exchange after fusion; (b) recoil exchange before fusion.
the final nucleus. We see that these two processes result in vastly different reaction amplitudes.

**Derivation of the Results.** The essential difference between the fusion process in free space and in a lattice is the availability of a catalyst, i.e., of a nearby lattice nucleus which serves as the third body needed to fulfill the requirements of energy-momentum conservation. This mechanism is the same for any choice of fusing and catalyzing nuclei. In order to fix the language we consider the case of radiationless \( p + d \) fusion in a Pd lattice. Then we have

\[
p + d + \text{Pd} \rightarrow ^3\text{He} + \text{Pd} + Q
\]

where the fusion releases the energy \( Q = 5.4 \text{ MeV} \). Inasmuch as the system is extremely nonrelativistic, the description can be reduced to solving appropriate Schrödinger equations in position space, after the energy conservation implied by the Feynman graphs has been taken into account. We now give the details.

The nonrelativistic first-order transition matrix element for both Fig. 1a and Fig. 1b has the form

\[
M = \int \psi_f^*(\vec{x}_1) \psi_i(\vec{x}_1) \left[ \frac{1}{|\vec{x}_1 - \vec{x}_2|} \right] \psi_f^*(\vec{x}_2, \vec{x}_3) \psi_i(\vec{x}_2, \vec{x}_3) \, d^3x_1 d^3x_2 d^3x_3
\]

The difference between the reaction in Fig. 1a and the reaction in Fig. 1b lies in the form of the wave functions for particles 2 and 3, i.e., \( \psi_f(\vec{x}_2, \vec{x}_3) \) for the initial state, while \( \psi_i(\vec{x}_2, \vec{x}_3) \) for the final state and \( \psi_i(\vec{x}_1) \) and \( \psi_f(\vec{x}_1) \) for particle 1 are the same for both cases. Thus, for reaction 1a, \( \psi_f(\vec{x}_2, \vec{x}_3) \) is that of a highly excited (5.4-MeV) \(^3\text{He} \) nucleus, while the reaction 1b, \( \psi_i(\vec{x}_2, \vec{x}_3) \) is the solid-state wave function of the \( p \) and \( d \) in the hydride, while \( \psi_i(\vec{x}_2, \vec{x}_3) \) is that of the \(^3\text{He} \) ground state in both cases. As becomes clear below, it is the fact that the fusion leads directly to the ground state of \(^3\text{He} \) which is essential in achieving penetration of the Coulomb barrier.

We begin with the angular parts. In view of the fact that the required momentum transfer \( |q| \rightarrow \text{fm}^{-1} \) while the interparticle distances are \( \sim \text{fm} \), we see that \( l \) values up to \( \sim 10^4 \) are involved, while the \(^3\text{He} \) ground state has \( l = 0 \). Alternatively one can say that here a description by ray (rather than wave) optics is called for. This implies that in order for the reaction to occur the Pd, \( p \), and \( d \) nuclei must be lined up within an accuracy \( \sim \text{fm} \). Either way, the angular overlap will yield a factor \( (\text{fm}/\text{A})^2 \sim 10^{-8} \) in the case of uncorrelated angles.

We now proceed to compute the wave function for the relative motion of particles 2 and 3. Introducing now the notation \( r \) for the distance between particle 2 and particle 3 we see that in order to evaluate the matrix element (2), we need to know \( \psi(r) \mid r = |\vec{x}_2 - \vec{x}_3| \), where \( \psi(r) \) obeys the radial part of the Schrödinger equation (here \( E \) is not an eigenvalue in view of the energy sharing with particle 1) \( (\hbar = c = 1) \)

\[
\left( -\frac{1}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + V(r) - E \right) \psi(r) = 0
\]

where \( \mu^{-1} = m_1^{-1} + m_2^{-1} \) is the reduced mass. Furthermore, \( V(r) \) is the Coulomb potential for \( r \rightarrow \infty \) and the effective nuclear potential \( 0 < r < \rho \), where \( \rho \) is the range of the nuclear forces. To treat that region one may introduce an effective one-body potential which can be chosen to yield agreement between theory and experiment over a given energy range—as is commonly done in the optical model of nuclear physics. We denote the solution of (4) for \( r < \rho \) by \( \psi_{\text{in}}(r) \).

We continue by considering the region \( r > \rho \), i.e., the Coulomb barrier. The two (not normalized) WKB solutions are

\[
\psi_{\text{reg}}(r) = e^{X(r)}
\]

\[
\psi_{\text{irreg}}(r) = e^{-X(r)}
\]

with

\[
X(r) = \int_0^r dr' \kappa(r')
\]

and

\[
\kappa(r') = \left\{ 2\mu \left[ \frac{l(l+1)}{r'^2} + \frac{e^2}{r'} - E \right] \right\}^{1/2}
\]

As evident from (7), \( \kappa(r') \) is a very slow function of \( E \). The solution in the Coulomb region then is

\[
\psi_{\text{Coul}}(r) = A\psi_{\text{reg}}(r) + B\psi_{\text{irreg}}(r)
\]

In order to determine the amplitudes \( A \) and \( B \) one must match wave functions and derivatives of the Coulomb region and the nuclear region at the nuclear radius \( r = \rho \). Within the accuracy of the WKB method we have, from (4), (5), and (6),

\[
\psi_{\text{reg}}(r) = \kappa(r) e^{X(r)}
\]

\[
\psi_{\text{irreg}}(r) = -\kappa(r) e^{-X(r)}
\]

We now are prepared to write down the Coulomb wave function and its derivative at \( r = |\vec{x}_{23}| \) needed in