STEADY HYDROMAGNETIC FLOW BETWEEN TWO POROUS CONCENTRIC CIRCULAR CYLINDERS

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The problem considered here is to study the MHD effects on the steady flow of an incompressible viscous conducting fluid through two concentric porous non-conducting infinite circular cylinders, rotating in various ways with uniform angular velocities in presence of a radial magnetic field. It is supposed that the rate of suction at the inner cylinder is equal to the rate of injection at the outer.

NOMENCLATURE

\( H \) magnetic field vector
\( B_0 \) magnetic induction vector
\( p \) pressure
\( \mu_e \) magnetic permeability
\( q \) density
\( \mu \) co-efficient of viscosity
\( v \) kinematic co-efficient of viscosity
\( \sigma \) conductivity of the medium
\( S \) suction parameter
\( \Omega_1 \) uniform angular velocity of the inner cylinder
\( \Omega_2 \) uniform angular velocity of the outer cylinder
\( \omega \) parameter due to magnetic field, \( \omega^2 = \sigma B_0^2 \mu \)
\( m \) constant
\( n \) parameter, \( n^2 = m^2 + \omega^2 \)
\( a, b \) radii of the co-axial cylinders
\( A, B \) constants for \( n \neq 0 \)
\( A_0, B_0 \) constants for \( n = 0 \)

1. INTRODUCTION

The theory of steady hydromagnetic flow through the ducts of rectangular cross-section in presence of a transverse uniform magnetic field has been presented by Shercliff [1], and Chang and Lundgen [2]. Edward [3] has studied the same problem when the duct section is an annulus and the impressed field is radial or circular. The problem of impulsive motion of a viscous liquid through two concentric circular cylinder in presence of a radial field has been presented by Singh and Rizvi [4]. Again the problems of impulsive motion of a viscous liquid contained between two porous concentric circular cylinders in presence of both radial and axial magnetic
fields have been investigated by Singh [5]. Mahapatra [6] has investigated the problems of unsteady motion of a viscous conducting liquid between two porous non-conducting infinite concentric circular cylinders rotating with various angular velocities for some time in presence of a radial field. The object of the present paper is to study the steady motion of an incompressible viscous conducting liquid between two porous non-conducting infinite concentric circular cylinders rotating in various ways with prescribed uniform angular velocities in presence of a uniform radial magnetic field.

2. FORMULATION OF THE PROBLEM

We consider here two concentric porous non-conducting infinite circular cylinders of radii \( a, b \) \( (b > a) \) and let an incompressible viscous conducting liquid be flowing steadily between them. Then introducing cylindrical co-ordinates \((r, \theta, z)\) let the components of velocity be denoted by \((u, v, w)\) and let the common axis of the cylinders coincides with the \(z\)-axis. Now let the magnetic field \(H_0\) be applied in the radial direction. The motion is rotationally symmetric and two dimensional and hence the axial component of velocity, \(w = 0\) and also the derivatives of transverse velocity with respect to \(\theta\) and \(z\) vanish. The induced electric and magnetic field are also neglected. Then the modified governing equations of steady motion and the equation of continuity are

\[
\begin{align*}
\frac{\partial u}{\partial r} - \frac{v^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + v \left( \nabla^2 u - \frac{u}{r^2} \right) \\
\frac{\partial v}{\partial r} + \frac{uv}{r} &= v \left( \nabla^2 v - \frac{v}{r^2} \right) - \frac{\sigma B_0^2}{\rho} \frac{v}{r^2} \\
\frac{d}{dr} (ru) &= 0
\end{align*}
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}, \quad B_0 = \mu_e H_0.
\]

In these equations the rationalized M.K.S. units have been used.

Also in this paper, the motion of the conducting fluid contained between these two concentric infinite circular cylinders will be considered in the following cases:

(I) By rotating both the cylinders with uniform angular velocities \((\Omega_1, \Omega_2)\).

(II) The inner cylinder is at rest whilst the outer cylinder is rotated with uniform angular velocity \(\Omega_2\).

(III) The outer cylinder is at rest whilst the inner one is rotated with uniform angular velocity \(\Omega_1\).