ANTISYMMETRIC-TENSOR GAUGE THEORY AND POLYNOMIAL INVARIANTS OF LINKS*)

B. BRODA†)

Arnold Sommerfeld Institute for Mathematical Physics,
Technical University of Clausthal,
Leibnizstraße 10, D-W-3392 Clausthal-Zellerfeld, FRG

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Non-abelian (topological) gauge theory of antisymmetric tensor field, so-called \(BF\)-theory, is analysed from the point of view of its applications to description of topological invariants of higher-dimensional links. Topological matter multiplets, playing the role of physical observables, are introduced to measure linking phenomena in the target space of arbitrary dimension. The whole theory, as a highly on-shell reducible gauge system, is quantized in the framework of the formalism of Batalin and Vilkovisky. A monodromy matrix, giving rise to a skein relation, is derived in a manifestly covariant path-integral way. A relationship with quantum groups is also mentioned.

It is well-known that (three-dimensional) Chern-Simons theory can be successfully applied to the derivation of polynomial invariants of knots and links \[1\]. Other well-defined topological theory is so-called \(BF\)-theory, i.e. non-abelian gauge theory of antisymmetric tensor field \[2\]. We aim at showing that \(BF\)-theory can yield polynomial invariants of two-component links in a space of arbitrary dimension \(d\). Some other attempts in this direction have already been suggested in literature \[3\], but we will follow a more general scheme proposed in \[4\]. Namely, we will define \(BF\)-theory together with some matter multiplets measuring linking phenomena in the target space, and next we will quantize the theory in the framework of the formalism of Batalin and Vilkovisky \[5\]. Using the Stokes theorem and a path-integral representation of the partition function, we will derive, in a non-perturbative and manifestly covariant manner, a monodromy matrix giving rise to a corresponding skein relation. We conclude the work relating our approach to the quantum group one.

The classical action of \(BF\)-theory is defined as

\[
S_{BF}^{cl} = \frac{1}{\lambda} \int_{S} \text{Tr}(B \wedge F),
\]

(1)

where \(\lambda\) is a (complex) coupling constant, \(S\) is the \(d\)-dimensional sphere, \(B\) is the Lie-algebra valued \((d - 2)\)-form, \(B = T^a B^a\), \(F\) is the curvature two-form, and

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†) Humboldt fellow
†) Permanent address: Department of Theoretical Physics, University of Łódź, Pomorska 149/153, PL-90-236 Łódź, Poland.
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\[ \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}. \]

The action of the matter fields consists of the two parts:

\[ S_{\text{cl}} = \frac{1}{2} \int_{\mathcal{K}} (\bar{\Theta} d_A \Omega + d_A \bar{\Omega} \Theta + \bar{\Theta} B \Theta), \quad (2a) \]

where \( \Theta \) and \( \bar{\Theta} \) are the zero-forms, \( \Omega \) and \( \bar{\Omega} \) are the \((d-3)\)-forms (all the forms are in the irrep \( R_1(G) \)), \( d_A \) is the exterior covariant derivative, and \( \mathcal{K} \) is the \((d-2)\)-dimensional closed submanifold imbedded in \( S \);

\[ S_{\eta} = \frac{1}{2} \int_{\mathcal{C}} \bar{\eta} d_A \eta, \quad (2b) \]

where \( \eta \) and \( \bar{\eta} \) are the zero-forms in the irrep \( R_2(G) \), and \( \mathcal{C} \) is the (one-dimensional) loop imbedded in \( S \). Then the classical action of the whole theory is

\[ S_{\text{cl}} = S_{\text{BF}}^{\text{cl}} + S_{\eta}^{\text{cl}} + S_{\bar{\eta}}^{\text{cl}}. \quad (3) \]

In general case, the action (3) enjoys four kinds of local gauge symmetries:

1. Ordinary gauge symmetry;
2. \( B \)-symmetry, which is \((d-3)\)-stage on-shell reducible,

\[ \delta_2 B = \frac{1}{2} \lambda \sigma_2, \quad \delta_2 \Omega = -\frac{1}{2} \sigma_2 \Theta, \quad \delta_2 \bar{\Omega} = -\frac{1}{2} \bar{\Theta} \sigma_2, \quad (4) \]

where \( \sigma_2 \) is the \((d-3)\)-form in \( R_{\text{Adj}}(G) \); (3) \((d-4)\)-stage on-shell reducible "matter" gauge symmetry of \( \Omega 

\[ \delta_3 B = \frac{1}{2} \lambda \delta(\mathcal{K}) \wedge \bar{\Theta} t_1 \sigma_3, \quad \delta_3 \Omega = -\frac{1}{2} d_A \sigma_3, \quad (5) \]

where \( \sigma_3 \) is the \((d-4)\)-form in \( R_1(G) \), and \( \delta(\mathcal{K}) \) is the Dirac-delta two-form; (4) \((d-4)\)-stage on-shell reducible "matter" gauge symmetry of \( \bar{\Omega} 

\[ \delta_4 B = \frac{1}{2} \lambda \delta(\mathcal{K}) \wedge \bar{\sigma}_4 t \Theta, \quad \delta_4 \bar{\Omega} = -\frac{1}{2} d_A \bar{\sigma}_4, \quad (6) \]

where \( \bar{\sigma}_4 \) is the \((d-4)\)-form in \( R_1(G) \).

Covariant quantization of on-shell reducible gauge systems should be approached by means of the Batalin-Vilkovisky procedure. The final result of such a procedure is a covariant path-integral representation of the partition function \( Z \). The problem is essentially solved if one succeeds in finding the solution \( S \) of the master equation \[ (S, S) = 0. \quad (7) \]

In our case,

\[ S = S_{\text{BF}} + S_{\eta} + S_{\bar{\eta}}, \quad (8) \]

with

\[ S_{\text{BF}} = \frac{1}{\lambda} \int_S \text{Tr}(b \wedge f), \quad (9a) \]