MODEL-INDEPENDENT RESULT FOR THE IMAGINARY PART OF FORWARD $\pi$He$^4$ SCATTERING AMPLITUDE IN UNPHYSICAL REGION

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Evaluation of contributions from dispersion integrals to the real part of forward $\pi$He$^4$ scattering amplitude is reconsidered. By application of a generalized mean-value theorem to the dispersion integral from the lowest branch point to the elastic threshold, a model-independent conclusion on the imaginary part of forward $\pi$He$^4$ scattering amplitude in unphysical region is drawn.

1. INTRODUCTION

Generally, there are two families of forward dispersion relations of the elastic scattering of two strongly interacting particles. They can be classified according to the relative position of an elastic threshold $\omega_{el}$ to the first inelastic or eventually anomalous threshold $\omega_0$ on the physical sheet of Riemann surface.

The first family includes the forward dispersion relations without the unphysical cut, i.e. the following inequality $\omega_0 > \omega_{el}$ between the corresponding thresholds is fulfilled. In this case the forward dispersion relations can be in principle used to relate the real part of forward scattering amplitude with experimental data on total cross section without any admixture of a model dependence at all energies. In case that an eventual appearance of pole terms in unphysical region were misleading (provided that one does not want to utilize the method of determination of corresponding residues based on the use of analyticity in cos $\theta$ plane [1]) we stress that by using symmetric (under crossing) amplitudes the noticeable contributions of pole terms is suppressed [2].

The forward dispersion relations containing an unphysical cut (the inequality $\omega_0 < \omega_{el}$ holds in this case) represent the second family. The low energy behaviour of a real part of amplitude evaluated by means of such dispersion relations depends to some extent on models used for the imaginary part in unphysical region.

There is another characteristic feature related with the appearance of an unphysical cut which is inherent for binary processes belonging to the same family. We have in mind the singular nature of total cross section in elastic threshold (known in nuclear physics as $1/v$ law [3], $v$ is the velocity of an incident particle in laboratory system) in the case $\omega_0 < \omega_{el}$. If no unphysical cut is present (i.e. $\omega_0 > \omega_{el}$) the total cross section takes nonnegative finite value. This feature can be simply understood as a direct consequence of the optical theorem, the parametrization of partial wave amplitudes and their threshold behaviour. Really, for all processes belonging to the first family, the imaginary part of forward scattering amplitude at the elastic threshold is zero. This will be fulfilled by taking the limit $k_L \to 0$ ($k_L$ is the incident laboratory system momentum) in the optical theorem for any nonnegative finite value of total cross section. On the other hand, the imaginary part of forward scattering amplitude of processes from the second family is equal to the imaginary part of the complex s-wave scattering length which is always a positive real number. Any finite real positive number will be reached by taking the limit $k_L \to 0$ in the optical theorem only if $\sigma_{tot} \to +\infty$ at elastic threshold.

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In this paper we shall be interested in the forward $\pi^4$ scattering which belongs to the second family. The corresponding forward dispersion relation will contain the contribution from an unphysical cut because there the channel $\pi^4 \rightarrow TN$ (T means He$^3$ or H$^3$ and N represents the nucleon) is open, which is responsible for the lowest of all existing branch points in $\pi^4$ scattering process.

The correct evaluation of $\pi^4$ forward dispersion relation and a comparison of obtained results with experimental data, strictly speaking, can be carried out only if experimental data on $\sigma_{\text{tot}}$ and Re$f(\omega)$ for both positive and negative pions at the same energies and in a sufficiently wide energy interval will be available. At present we have the situation when these quantities were measured at only very few points for both charged pions simultaneously and the remainder of experimental data exists either for positive or negative pions only. Despite of this it seems to us that some interesting results can be obtained also under the present experimental situation. This opinion was recently confirmed [2, 4, 5] to some extent.

The first results on the real part of forward $\pi^4$ scattering amplitude by a dispersion method were obtained by ERICSON and LOCHER [2]. At that time only four experimental points on the real part were available and nothing could be said about the agreement between experimental and theoretical results.

The authors of paper [6] were discussing the inconsistency of the dispersion relation prediction [2] with new experimental data. This incited one of us (S.D.) to analyze in detail the calculation of the real part [4] and to look for the most probable cause why it was impossible to get consistency between the experimental data on the real part and total cross section through forward dispersion relation.

The result of paper [4] is the following. To get a better agreement of the calculated real part with experiment, one has to shift the maximum of total cross section in resonant region to lower energies and to higher values. This prediction was later confirmed experimentally by WILKIN et al. [7] and by inclusion of the last data the agreement with experimental values of the real part was improved [5].

Recently again new experimental data (four points) on real part have appeared [8] and three of them at higher energies seem to be inconsistent also with the newest dispersion predictions [5].

The aim of the present paper is to investigate to what extent one can expect that the imaginary part of forward $\pi^4$ scattering amplitude in unphysical region can improve the situation. Because as it will be seen later neither physical (here we have experimental data on total cross section) nor asymptotical regions can be responsible for such changed behaviour of the real part.

2. EVALUATION OF DISPERSION INTEGRALS

The once-subtracted forward dispersion relation for symmetric elastic $\pi^4$ scattering amplitude in laboratory system takes the form [2, 4, 5]

$$(1) \quad \text{Re} f(\omega) =$$

$$= \text{Re} f(\omega_s) + \frac{2(\omega^2 - \omega_s^2)}{\pi} \int_{\omega_{TN}}^{1} + \int_{17.14}^{17.14} + \int_{17.14}^{\infty} \frac{\omega' \text{Im} f(\omega')}{(\omega'^2 - \omega_s^2)(\omega'^2 - \omega^2)} d\omega'$$

where units $\hbar = c = \mu = 1$ are used, $\mu$ is the mass of pion, Re$f(\omega_s)$ is a subtraction constant and $\omega_{TN} \approx 0.143[\mu]$.

It will be seen in section 3 that one has to choose the subtraction constant at the value $\omega_s$ from the physical region unlike the papers [2, 4, 5]. Assuming that Re$f(\omega)$