BARYONS AND NUCLEI AS SKYRMIONS *)

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The topological soliton model, including its various extensions, is reviewed. The model is very compact and involves only small number of Lagrangian parameters which in principle are calculable from QCD. It provides a nice tool for describing the structure of baryons.

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1. Introduction

The main problem in the theoretical description of the structure of nuclei is the apparent conflict between the empirically established fact that nuclei are weakly bound systems of nucleons which interact by means of exchanging mesons and the QCD-based theoretical view of baryons and nuclei as strongly bound systems of quarks and gluons. One way of overcoming this problem is to try to construct effective hadron field theories that approximate those features of QCD that are relevant at low energies, and which can be linked to the observable meson and baryon fields. The key question is then to which extent the dynamical content of QCD can be described by such an effective hadron (meson) field theory.

Some important steps towards answering this question as far as the structure of baryons is concerned were the following: (1) In 1961 Skyrme found that it is possible to construct simple chirally symmetric meson field theories, which have "soliton" solutions that can be quantized as fermions [1], (2) In 1978 Witten noted that in the limit of a large colour number QCD is equivalent to a nonlinear chiral meson field

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theory and that in this case the baryons have to be recovered as soliton solutions [2]. Thus Witten's result provided a dynamical justification for Skyrme's original model, and moreover showed in which sense it can be viewed as an approximation to QCD in the limit of a large colour number. A simple explanation for this observation can be obtained by considering Feynman diagrams of quark lines. In the $N_c \to \infty$ limit the contributions of the non-planar Feynman diagrams vanish and the quark-antiquark lines in the remaining planar diagrams can be described pairwise as bosons — i.e. mesons [3].

In principle, the nonlinear meson field theory that approximates QCD contains an infinite number of meson fields. The reason that this approach is nevertheless useful is the fact that only the light mesons are of numerical significance in the regime of low energies and relatively long range interactions that characterize nuclear physics. Because of this the general meson field theory may be truncated to one formed of only the lightest meson fields (e.g. the pion and vector meson fields).

The fact that the effective meson field theory has to have chiral symmetry is — besides the fact that this is a symmetry of the QCD in the light (u, d) flavour sector — justified by the empirical fact that the strong interactions are independent under isospin rotations ($SU(2)_V$) and independent of intrinsic parity ($SU(2)_A$). This $SU(2)_V \times SU(2)_A$ invariance is referred to as chiral invariance. Associated with this symmetry are two Noether currents: V^μ, A^μ. The vector current V^μ is conserved, whereas the axial current A^μ is only "almost" conserved — its divergence is proportional to the small mass of the light quarks.

2. The non-linear σ-model

The simplest chiral meson theory involves only the isotriplet pion fields $\vec{\pi}$. The conserved vector current density operator then has the form

$$\vec{V}^\mu = \vec{\pi} \times \partial^\mu \vec{\pi} \,. \tag{2.1}$$

It is, however, not possible to form an axial current density operator that is bilinear in the pion fields. To construct a proper axial current one needs an additional scalar meson field σ:

$$\vec{A}^\mu = \sigma \partial^\mu \vec{\pi} - \vec{\pi} \partial^\mu \sigma \,. \tag{2.2}$$

The scalar field σ does not have to be a dynamical field, however, but can be a non-linear function of the spin field modulus $\vec{\pi}^2$. This is the basis of the non-linear σ-model [4].

The non-linear σ-model is the formally free field Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2, \tag{2.3}$$

in which the σ-field is defined by the chirally symmetric constraint

$$\sigma^2 + \vec{\pi}^2 = f_\pi^2 \,. \tag{2.4}$$