Görtler Vortices and Boundary-layer Transition

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Introduction

The effect of surface curvature on the instability of a laminar boundary layer due to two-dimensional disturbances is very small for the curvature likely to occur in practice. Thus the instability oscillations, the so-called Tollmien-Schlichting waves, are expected to behave on a concave or convex curved surface almost similarly as on a flat plate. However, the flow on a concave surface exhibits another instability under the influence of centrifugal pressure gradient, generating a three-dimensional system of alternating vortices with axes in the streamwise direction, as studied theoretically by Görtler [1]. The streamwise vortices, now called Görtler vortices, are predicted to come into existence at a Reynolds number much below that for the appearance of the Tollmien-Schlichting waves.

The generation of Görtler vortices was first pointed out by Liepmann [2, 3] as responsible for hastening the transition to turbulent flow in the boundary layer on a concave surface. The traces of the vortices were also visualized by Gregory and Walker [5] as china-clay streaks on a concave flap surface. However, no direct experimental verification had been made until the junior author [9, 10] identified the vortices observed in the laminar boundary layer on a concave surface with those predicted by theory. In particular, the observed values of spanwise wavelength of the vortices were located within the region where the disturbances were to be amplified according to theory. Moreover, the distribution of amplitude across the boundary layer and the rate of amplification in streamwise direction were found to agree well with those predicted by Görtler’s theory and the modified theory due to Smith [4], respectively. A similar experimental verification of Görtler vortices has subsequently been made by Wortmann [12] utilizing flow visualization technique in water.

Since the Görtler vortices are amplified only moderately in most practical cases, they are considered as mainly responsible for making the boundary layer three-dimensional by redistributing the streamwise momentum. It is thus expected that the transition in the boundary layer on a concave surface is somewhat similar to that in the boundary layer on a flat plate with pre-existing streamwise vortices, or with spanwise variation in boundary-layer thickness. This was confirmed to a certain extent by the previous investigation [10, 11], in which the development of controlled twodimensional

1) On leave from Tokyo.
disturbances was traced in the boundary layer on a concave surface. However, some hesitation was felt in arriving at a final conclusion, mainly because of the insufficient understanding of the transition in a boundary layer with spanwise variation in thickness, which has only recently been elucidated by the detailed measurements of Komoda [13].

Quite recently, therefore, additional measurements have been carried out in a boundary layer on one of the concave surfaces used in the previous investigation [10], with a view to confirming and supplementing the results obtained previously. Based primarily on the results of these measurements, the paper presents the current interpretation of the boundary-layer transition in the presence of Görtler vortices. The authors wish to acknowledge the kind consideration of Professor H. Sato, University of Tokyo, for making the additional measurements possible.

**Görtler Vortices Naturally Generated on a Concave Surface**

Measurements were made on the boundary layer on a concave curved plate with a radius of curvature \( r \) of 300 cm, mounted in a wind tunnel of 60 cm by 60 cm working section. The wall opposite to the test surface was adjusted to make the free-stream velocity \( U_0 \) outside the boundary layer uniform within the accuracy of measurements. The reference axes were taken in such a way that \( x \) was measured along the plate from the leading edge in the streamwise direction, \( y \) in the direction normal to the plate, and \( z \) in the spanwise direction normal to \( x \) and \( y \). Most measurements were made at \( U_0 = 7 \text{ m/s} \), where the free-stream turbulence level was about 0.05 per cent.

The experimental arrangement was almost the same as in the previous investigation [10], except for a slight difference in the curved plate. The plate used for the new measurements had a leading-edge portion of plane surface 40 cm long, whereas the plate for the previous measurements was curved immediately from the leading edge. In Figure 1 is reproduced a typical variation of streamwise mean velocity \( U \) obtained by traversing a hot-wire probe in the \( z \) direction at a fixed height \( y \) from the plate in the previous measurements [10]. The nearly periodic spanwise variation of velocity is regarded as produced by the system of Görtler vortices superposed on the Blasius flow. The spanwise wavelength was about 2 cm. In this case the transition was located at about \( x = 90 \text{ cm} \), where the boundary layer thickness increased considerably.

In the new measurements the wavelength of spanwise variation of mean velocity was again about 2 cm. The transition was located at about \( x = 110 \text{ cm} \), apparently delayed by the addition of a plane leading edge. However, the transition Reynolds number based on \( U_0 \) and the boundary-layer displacement thickness \( \delta^* \) was found to remain almost the same.

Values of the displacement-thickness Reynolds number \( R_{\delta^*} \) at transition are shown in figure 2 as a function of \( \delta^*/r \), the negative sign of \( r \) corresponding to the convex surface. Most of the experimental points are due to Liefmann [2, 3], only a few having been added by the present authors. Since the definition of the transition location is different between the two experiments, no great importance may be placed on determining the parameter characterizing the transition. For practical purposes, however, it may safely be said that the transition is correlated with a certain critical value of the Görtler parameter \( G = R_{\delta^*}(\delta^*/r)^{1/2} \), the value increasing as \( \delta^*/r \) increases.