Heat Transfer in a Growing Vapor Film

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The present note analyzes the unsteady transfer of energy by conduction, convection and radiation in a growing vapor film. This subject is of basic interest and also has important applications in phenomena associated with reactor transients and with quenching.

The specific problem considered is the one-dimensional growth of a vapor film of uniform thickness. The analysis of this problem is made difficult due to the highly nonlinear nature of the radiation contribution, the coupling between the basic equations, and the effects of variable properties. In an earlier study, Hamill and Bankoff [1] obtained results for the rate of growth of a vapor film omitting the effects of radiation and assuming constant properties.

The vapor region between the surface and the interface is assumed to be planar and free of end effects (see Fig. 1). Viscous dissipation may be neglected and the pressure assumed constant. Under these conditions the equations of continuity and energy are given by

\[ \frac{\partial q}{\partial t} + \frac{\partial (q v)}{\partial x} = 0, \]  

\[ q c_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right] = \frac{\partial}{\partial x} \left[ h(T) \frac{\partial T}{\partial x} \right] - \frac{\partial q_R}{\partial x}, \]  

where \( q_R \) is the radiative flux. For this problem the optical depths are small (much less than 0.1) so that the radiation contribution is restricted to the optically thin region. We therefore use the following relation for the divergence of the radiative flux [2]:

\[ \frac{\partial q_R}{\partial x} = 4 \sigma K_p(T) T^4 - 4 \sigma K_m(T, T_w) T_w^4, \]  

where \( K_m \), the modified Planck mean absorption coefficient, may be approximated by [3]:

\[ K_m(T, T_w) = K_p(T_w) \frac{T_w^4}{T}. \]  

Thus, the energy equation becomes

\[ q c_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right] = \frac{\partial}{\partial x} \left[ h(T) \frac{\partial T}{\partial x} \right] - 4 \sigma \left[ K_p(T) T^4 - K_p(T_w) \frac{T_w^4}{T} \right]. \]  

Figure 1
Physical model.

The liquid is considered to be initially at the saturation temperature \( T_s \), and the solid surface is assumed to be instantaneously increased to the temperature \( T_w \). The boundary
conditions are given by
\[ T(O, t) = T_w, \quad T(\delta, t) = T_s \] (6)
where \( \delta \), the thickness of the vapor film, is an unknown function of time. Thus, another boundary condition is required and this is obtained by equating the rate at which energy is transferred to the liquid to the rate at which energy must be supplied to evaporate the liquid, viz.
\[ -k \frac{\partial T}{\partial x} \bigg|_{x-\delta} + q_R \bigg|_{x-\delta} = \lambda \frac{d}{dt} \left[ \int_0^\delta q \, dx \right]. \] (7)

For the small optical depths that are characteristic of this problem, the radiative heat flux may be accurately approximated by the non-participating relation so that
\[ q_R \bigg|_{x-\delta} = \varepsilon_w a (T_w^4 - T_s^4) \] (8)
where the liquid surface is taken to be a black body [4]. Sparrow [4] has shown that for the steady state problem the maximum error due to the omission of the radiative participating effects of the vapor is 3 percent.

The problem is considerably simplified by introducing the Lagrangian coordinate, \( m \), which equals the mass of the vapor contained between the plate and the distance \( x \) from the plate, per unit of area:
\[ m = \int_0^x q \, dx. \] (9)
The basic equation then becomes
\[ \left( \frac{\partial \theta}{\partial t} \right)_m = \frac{1}{c_p} \frac{\partial}{\partial m} \left[ k \frac{\theta}{m} \frac{\partial \theta}{\partial m} \right] + \frac{4 \sigma T_s^4}{c_p q} \left[ K_p(T) \cdot (1 + \theta)^4 - K_p(T_w) \frac{T_w}{T_s} \right] \] (10)
where
\[ \frac{T}{T_s} = 1 + \theta. \] (11)

The energy balance at the interface is given by
\[ -k_s \varepsilon_s T_s \left( \frac{\partial \theta}{\partial m} \right)_{m=M} + \sigma T_s^4 \left[ \left( \frac{T_w}{T_s} \right)^4 - 1 \right] = \lambda \frac{dM}{dt} \] (12)
where \( M \) is the total mass of the vapor per unit of area:
\[ M = \int_0^{\delta(t)} q \, dx. \] (13)

Before we can begin to solve this problem the Planck mean absorption coefficient, \( K_p \), and the thermal conductivity, \( k \), must be prescribed as functions of the temperature. From the results of ABO-ROMIA and Tien [5] (also see HOTTEL and SAROFIM [6]) we note that the relation \( K_p \cdot T = \text{constant} \) is a good approximation for water vapor. Also, the relation \( K = \text{constant} \cdot T \) is in good agreement with thermal conductivity data for water vapor [7]. We shall therefore use these relations in the determination of the temperature profiles for growing water vapor films.

To obtain the temperature distribution within the vapor film as a function of time and position we must solve the non-linear problem formulated above. We shall solve this problem approximately using integral methods (GOODMAN [8], VON KÁRMÁN and POHLHAUSEN [9]). The energy equation, Equation (10), is multiplied by \( dm \) and integrated over the mass of the vapor. The resulting equation is called the heat balance integral and the temperature is required to satisfy this relation but not the original equation, Equation (10).