PLASMA ACCELERATION IN A WAVE WITH VARYING FREQUENCY

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The averaged velocity of a test particle and the averaged velocity of a plasma in an electromagnetic wave packet with varying frequency (e.g., a radiation pulse from pulsar) is derived. The total momentum left by the wave packet in regions of plasma inhomogeneity is found. If the plasma concentration is changing due to ionization, the plasma may be accelerated parallelly or antiparallelly to the direction of the wave packet propagation, which is relevant for a laser induced breakdown in gas.

1. INTRODUCTION

As is well known, quasi-monochromatic electromagnetic wave packets may accelerate matter due to average forces proportional to a temporal and spatial gradient of wave amplitude, see e.g. [1, 2]. In [3], the equation governing the averaged velocity of a charged particle is given for the case of an electric and magnetic field with temporal and spatial variations of the amplitude, frequency and wave vector. The pulses of radiation coming from pulsars are examples of such waves. By using the results of [3] for \( N < 1 \) (\( N \) being the refraction index), the expression for the average velocity of the charged particle in a wave packet with varying frequency has been derived in [11].

The object of the present paper is to find the average velocity of charged particles by integrating the equation for the temporal change of the averaged velocity [3]. We will omit the restrictive assumption [1] concerning the refraction index magnitude.

2. BASIC EQUATIONS

Let us consider an electromagnetic wave packet with the electric field \( E_0(z, t) \cdot \exp [i \Psi(z, t)] \) polarized along the x-coordinate, \( E_0 \) being the slowly varying amplitude and \( \Psi \) being the Eikonal function. The case of an arbitrarily polarized wave would be treated analogically. Throughout the paper we will assume the validity of the geometrical optics approximation for waves with varying frequency, see e.g. [4]. The wave packet propagates in a cold inhomogeneous plasma parallelly to the z-coordinate, which is chosen in the direction of the plasma concentration gradient. Denoting \( \partial \psi/\partial z \) by \( k(z, t) \) and \(- \partial \psi/\partial t \) by \( \omega(z, t) \), we may write the following equation:
tions [4] for $k$, $\omega$ and $E_0$:

$$
\begin{align*}
(1) & \quad k \frac{\partial E_0}{\partial z} + \frac{\omega^2}{c^2} \frac{\partial E_0}{\partial t} + \frac{E_0}{2} \left( \frac{\partial k}{\partial z} + \frac{1}{c^2} \frac{\partial \omega}{\partial t} \right) = 0, \\
(2) & \quad k^2 - \frac{\omega^2}{c^2} + \frac{\omega_p^2}{c^2} = 0,
\end{align*}
$$

$\omega_p(z, t)$ being the electron plasma frequency. We study only the simple case, in which the time dependence of $\omega_p$ is caused by ionization. The dispersion equation (2) implies

$$
\begin{align*}
(3) & \quad k \frac{\partial k}{\partial z} - \frac{\omega}{c^2} \frac{\partial \omega}{\partial z} = - \frac{\omega_p}{c^2} \frac{\partial \omega_p}{\partial z}, \\
(4) & \quad k \frac{\partial k}{\partial t} - \frac{\omega}{c^2} \frac{\partial \omega}{\partial t} = - \frac{\omega_p}{c^2} \frac{\partial \omega_p}{\partial t}.
\end{align*}
$$

By using the relation

$$
\frac{\partial k}{\partial t} + \frac{\partial \omega}{\partial z} = 0,
$$

we obtain from (3)

$$
\begin{align*}
(5) & \quad \frac{\partial \omega}{\partial t} + c e^{1/2} \frac{\partial \omega}{\partial z} = \frac{\omega_p}{\omega} \frac{\partial \omega_p}{\partial t}, \\
(6) & \quad \frac{\partial \omega}{\partial t} + c e^{1/2} \frac{\partial \omega}{\partial z} = \frac{\omega_p}{\omega} \frac{\partial \omega_p}{\partial t},
\end{align*}
$$

where $\varepsilon = 1 - \omega_p^2/\omega^2$ is the dielectric permittivity of the cold plasma.

The averaged velocity $V_z$ of a particle with the mass $\mu$ and the electron charge $e$ is governed by equation [3]

$$
\frac{dV_z}{dt} = - \frac{e^2}{2\mu^2\omega^2} \left[ \frac{1}{2} \frac{\partial (E_0 E_0^*)}{\partial z} + \frac{\partial k}{\partial t} \frac{E_0 E_0^*}{\omega} \right] = F.
$$

The right side of (8) is simplified according to the assumption that the initial unperturbed particle velocity is zero. The terms proportional to $E_0^4$ are omitted, too.

3. THE AVERAGED VELOCITY AND MOMENTUM

If the temporal and spatial derivatives of plasma concentration $n$ are zero, the right sides of equations (3, 4, 6, 7) vanish. Using equations (1—7), we transform equation (8) to another form,

$$
\frac{dV_z}{dt} = - \frac{e^2}{2\mu^2\omega^2} \left[ - \frac{1}{2c e^{1/2}} \frac{\partial (E_0 E_0^*)}{\partial t} - \frac{E_0 E_0^*}{\omega} \frac{\partial \omega}{\partial t} - \frac{1}{c e^{3/2}} + \frac{E_0 E_0^*}{\omega} \frac{1}{c e^{1/2}} \frac{\partial \omega}{\partial t} \right],
$$