Propagation of Cylindrical Blast Waves in Magneto-Gas dynamics

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Introduction

Propagation of cylindrical blast waves in a plasma, under a constant axial current, has been studied by Greenspan [3], Greifinger and Cole [2], and Christer and Helliwell [1]. These authors have sought similarity solutions of the problem for a very strong instantaneous line explosion. Korobeinikov [4] has considered the problem of an explosion in a gas of constant density and pressure, in the absence of any current, and has assumed the existence of an initial uniform magnetic field in the axial direction. He has reduced the equations of motion in terms of two independent variables in suitable forms to effect numerical computations.

In the present paper, we consider a similar problem of the one of Korobeinikov [4]. We seek such solutions, which maintain their similarity forms, except at the shock surface, heading the disturbed region. After Sedov [5], we name such motions non-self-similar. The total energy of the disturbance is non-constant but can be made to increase at a slow rate. The solutions derived in closed forms are available for sufficiently long time and reduce to the ordinary non-magnetic case as a particular one. Effects of viscosity and heat conduction have not been taken into account.

Equations of the Problem and Boundary Conditions

The equations, governing the motion of the fluid, can be expressed as follows:

\[ \frac{\partial E}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r u I) = 0 , \]  

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \rho \frac{\partial \rho}{\partial r} + \frac{\rho u}{r} = 0 , \]  

\[ \frac{\partial H}{\partial t} + u \frac{\partial H}{\partial r} + H \frac{\partial u}{\partial r} + \frac{H u}{r} = 0 , \]  

\[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} = \frac{\gamma p}{\varrho} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) , \]  

where \( u, \rho, \varrho \) and \( H \) are the velocity, pressure, density and the axially-directed magnetic field at a distance \( r \) from the line of explosion at time \( t \); \( \gamma \) is the ratio of the specific heats and lies between 1 and 2.
Besides, we have put
\[ E = \frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1} + \frac{H^2}{8\pi} \]  
and
\[ I = E + p + \frac{H^2}{8\pi} . \]

The motion is supposed to be bounded on the outside by a cylindrical shock surface at \( r = R(t) \), which moves outward with velocity
\[ V = \frac{dR}{dt} . \]

If ahead of the shock the constant undisturbed pressure, density and the magnetic field parallel to the axis be \( p_\infty, \rho_\infty \) and \( H_\infty \) and those just behind the shock be \( p_n, \rho_n \) and \( H_n \), where also the velocity is \( u_n \), then we have the following from the usual shock conditions (Korobeinikov [4]):

\[ \frac{u_n}{V} = 1 - \frac{\rho_\infty}{\rho_n}, \]

\[ \frac{H_n}{H_\infty} = \frac{\rho_n}{\rho_\infty}, \]

\[ - \left(1 - \frac{\rho_\infty}{\rho_n}\right) + \frac{\rho_n}{\rho_\infty} \left(\frac{1}{\gamma} \frac{H_n^2}{H_\infty^2} + \frac{1}{2 M_0^2} \right) = \frac{1}{2 M_0^2} + \frac{1}{\gamma M^2}, \]

\[ - \frac{1}{2} \left(1 - \frac{\rho_\infty}{\rho_n}\right)^2 - \frac{\rho_\infty}{\rho_n} \left\{ \frac{1}{\gamma (\gamma - 1)} \frac{\rho_n}{\rho_\infty} \left(\frac{1}{M^2} + \frac{H_n^2}{H_\infty^2} \right) \right\} \]

\[ + \left(\frac{\rho_n}{\rho_\infty} \frac{1}{\gamma} \frac{M^2}{2 M_0^2} + \frac{1}{\gamma M^2} \right) \left(1 - \frac{\rho_\infty}{\rho_n}\right) \]

\[ + \frac{1}{\gamma (\gamma - 1) M^2} + \frac{1}{2 M_0^2} = 0, \]

where
\[ M^2 = \frac{q V^2 \infty}{\gamma \rho_\infty} \]  
and
\[ M_0^2 = \frac{4 \pi V^2 q_\infty}{H_\infty^2} . \]

Equations (8), (9) and (10) yield
\[ \frac{\rho_\infty}{\rho_n} = \frac{(\gamma - 1)}{(\gamma + 1)} \left[ \frac{\gamma}{\gamma - 1} \left(\frac{1}{2 M_0^2} + \frac{1}{\gamma M^2} \right) + \frac{1}{2} \right] \]

\[ + \left\{\left(\frac{\gamma}{\gamma - 1} \left(\frac{1}{2 M_0^2} + \frac{1}{\gamma M^2} \right) + \frac{1}{2}\right)^2 + \frac{(\gamma + 1)(2 - \gamma)}{4 M_0^2 (\gamma - 1)^2} \right\}^{1/2}. \]