THE TRANSVERSE VIBRATION OF SPINNING AEOLOTROPIC DISK

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The paper represents a general treatment of the problem of transverse vibration of a thin high speed rotating aeolotropic elastic circular disk of uniform thickness.

1. INTRODUCTION

It is well known that while a turbine disk is kept spinning by admission of impulsive steam flow over its blades, the disk is generally found to exhibit transverse vibration. A number of investigations [1 - 6] have been made in this regard to consider the transverse vibrations of rotating isotropic elastic disks.

In this connection it is worth noting that owing to the existence of high temperature and pressure of steam for long duration, it may happen that an isotropic turbine disk may fail to retain its elastic property same in every direction. Such an idea has led the author to consider the transverse vibration of a spinning aeolotropic elastic disk in the present paper.

2. THE PROBLEM

Consider a thin aeolotropic elastic disk of radius $a$ rotating rapidly about the normal axis with uniform angular velocity $\omega$. For a small displacement $w$ normal to the undisturbed central plane of the disk, the equation of transverse motion of an elementary mass $\rho r \delta \theta \delta r$, of negligibly small rigidity (LAMB & SOUTHWELL [1]), is

\[ \rho \frac{\partial^2 w}{\partial t^2} = \frac{1}{r} \frac{\partial}{\partial r} \left( Pr \frac{\partial w}{\partial r} \right) + \frac{Q}{r^2} \frac{\partial^2 w}{\partial \theta^2} \]

where $\rho$ is the density of the material, $P$ the radial and $Q$ circumferential (or, hoop) stress and $(r, \theta)$ the polar co-ordinates of a point with respect to the centre.

3. THE SOLUTION

3.1. Determination of $P$ and $Q$

The equation of equilibrium to be satisfied by the stresses is

\[ \frac{dP}{dr} + \frac{P - Q}{r} = -\rho \omega^2 r . \]
The transverse vibration of spinning aeolotropic disk

The appropriate strain-energy function, \( W \), is given by

\[
2W = c_{11}e_{rr}^2 + 2c_{12}e_{rr}e_{\theta\theta} + c_{22}e_{\theta\theta}^2
\]

where \( e_{rr} \), \( e_{\theta\theta} \), \( c_{11} \), \( c_{12} \) and \( c_{22} \) have the meanings indicated by Love [7].

Now,

\[
\begin{align*}
P &= c_{11} \frac{du}{dr} + c_{12} \frac{u}{r} \\
Q &= c_{12} \frac{du}{dr} + c_{22} \frac{u}{r}
\end{align*}
\]

where

\[
\frac{du}{dr} = e_{rr} \\
\frac{u}{r} = e_{\theta\theta}
\]

and \( u = \) radial displacement, independent of \( \theta \). On putting (3) in (2) we get

\[
\frac{d^2u}{dr^2} + c_{11} \frac{1}{r} \frac{du}{dr} - c_{22} \frac{u}{r^2} = -q\omega^2r.
\]

Let a particular integral of the last equation be

\[
u = Rr^3
\]

where \( R \) is a constant to be determined. On substituting (5) in (4) we get

\[
R = -\frac{q\omega^2}{9c_{11} - c_{22}}
\]

provided

\[
9c_{11} \neq c_{22}.
\]

To find the complementary function let us take \( u = A_1r^n \) which gives on substituting in (4)

\[
n = \pm \sqrt{\frac{c_{22}}{c_{11}}}.
\]

For a solid disk \( n \) must be positive, otherwise the displacement of the centre will be infinite, and henceforth by \( n \) we shall mean its positive value.

Hence

\[
u = A_1r^n + Rr^3
\]