Two types of invariant expansions of scattering amplitude based on group $O(3, 1)$ that can be considered as generalizations of the partial wave analysis are studied. They are shown to be identical with the Fourier transform of the modified partial waves, the modification being given by integrating or differentiating the partial wave. We search for a simple physical use of expansion coefficients, the so-called Lorentz amplitudes, and find the connection between the poles of Lorentz amplitude and the high-energy behaviour of the partial wave; the relation between the Lorentz amplitude and the position of the Regge poles is precisely formulated.

1. INTRODUCTION

The scattering amplitude $M_{\,fi}$ can be written in the form [1]

$$M_{\,fi} = \sum_n f_n(s, t, \ldots) F_n$$

where $F_n$ are constructed as invariants linearly dependent on wave amplitudes of all particles in question. The coefficients $f_n$ (invariant amplitudes) depend on the Mandelstam variables $s, t, \ldots$ and contain dynamical information about the scattering.

In the quantum electrodynamics we are in principle able to calculate $f_n$ using the perturbation theory [1]. In the theory of strong interactions, we formulate dynamical postulates directly for the structure of these functions [2]. Two-variable expansions can be treated as a method of the latter approach. The functions are expanded with the use of an orthogonal and complete set of functions and we have to study the properties of expansion coefficients, Lorentz amplitudes (abbr. LA), instead of invariant amplitudes (review article [3] contains deeper analysis and list of basic references).

The set of generalized harmonic functions is determined by the choice of a group and its reduction to subgroups. The desired output of such a formulation is a simplification of some problems (fitting of data [4], formulation of hypotheses) and new understanding of old principles (analyticity, invariance, crossing etc.). The physical input of re-formulation is the interpretation of the group.

In the case of the scattering $a + b \to c + d$ with zero spin, it is possible to choose the reduction $O(3, 1) \supset O(3) \supset O(2)$, corresponding to $S$-expansion, or $O(3, 1) \supset \supset O(2, 1) \supset O(2)$, expansion in $H$ system [5]. The physical reasons for this choice are [3] as follows:

R1) $O(3, 1)$ is a group of motion of one of the particles on the mass shell, which is interpreted as a motion of the point in the physical region of $s$-channel in the Mandelstam plane.
Two-variable expansions contain the partial wave expansions. The subgroups O(3) and O(2, 1) are also the little groups of the Poincaré group.

S- and H-expansions in crossed channels are analytic continuations of each other [6].

The formalism can be generalized for the case of non zero spins [7].

In our work we solve two problems in S- and H-system:

1) The formal simplification of the integral formulae.
2) The translation of asymptotics and poles of partial waves into the language of LA and vice versa.

The interest in point 1) is motivated by practical needs. Point 2) endeavours to connect the expansions with the usual theory of complex angular momentum.

2. GENERALIZED PW ANALYSIS

2.1. The formalism of S-expansion

According to Ref. [8] the following lemmas hold:

**Lemma 1.** Any function \( F(u) \) of fourvector \( u = (u_0, u_1, u_2, u_3) \)

\[
F(u) = \sum_{l=1}^{\infty} (2l + 1) a_l(a) P_l \cos \theta
\]

with

\[
a_l(a) = \frac{1}{\sqrt{(\cosh a)}} \int_0^\infty B_l(p) f_l(p) P_{-l/2+i/2}(\cosh a) \, dp
\]

(abbreviation of special functions see Ref. [9]), with inversion

\[
a_l(a) = \frac{1}{2} \int_{-1}^1 \sqrt{(\cosh a)} \, F(u) P_l \cos \theta
\]

and

\[
B_l(p) = f_l(-p) \int_0^\infty \sqrt{(\cosh a)} \, a_l(a) P_{-l/2+i/2}(\cosh a) \, \sin a \, da
\]

Here the abbreviation

\[
f_l(p) = ip(ip - 1) \cdots (ip - l) = \Gamma(ip + 1)/\Gamma(ip - l)
\]