Heat Transfer and Reynolds’ Analogy in a Turbulent Flow with Heat Release

By RAUL R. HUNZIKER, Washington, D. C., USA.

Introduction

PAT has found polynomial solutions of REYNOLDS’ turbulent flow equations expressing the time average axial velocity and correlation distributions in a circular pipe [1]. OSBORNE REYNOLDS was the first who inferred a dependency between the transference of heat and momentum in turbulent flow [2]. According to the ‘momentum transfer theory’ [2, 3] these distributions allow a consistent semiempirical determination of the eddy diffusivity for heat $\varepsilon_H = \alpha \varepsilon_M$ [4, 5, 6].

The Reynolds analogy for a circular pipe can be formulated departing from the axial Reynolds equation and the temperature distribution equation [3, 7]. It follows that the nondimensional temperature and velocity profile are identical if the fluid contains heat sources of intensity proportional to the constant axial component of the gradient of pressures and if the effective Prandtl number $Pr^* = Pr \alpha$ of the turbulent fluid is unity. Extensions of the analogy for $Pr^* \neq 1$ have been developed by PRANDTL [3] and TAYLOR [8] assuming a laminar layer close to the wall in which heat is transferred by conduction alone.

This separation of the field into laminar and turbulent regions was further improved by von KÁRMÁN, who introduced a transition of buffer layer in which both molecular and turbulent heat transactions were assumed [9, 10]. MARTINELLI has introduced the effect of molecular diffusivity in the turbulent core which is of special significance for the case of liquid metals [11].

Similar analysis allowing the representation of the radial distribution of temperatures at great distance from the pipe inlet have been developed by LYON [12], SEBAN and SHIMAZAKI [13].

The calculations based on the extensions of the analogy are usually compared however with measurements of heat transfer from fluids in which no sources of heat are present.

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2) Reed Research Inc., Chief Mathematics Branch, Research Division.

Present address: Convair, A Division of General Dynamics Corporation, San Diego, California.

3) Numbers in brackets refer to References, page 314.
The solution of the boundary value problem for the temperature equation can be constructed also for the case of a distribution of heat sources within the fluid. It will be shown that the Reynolds analogy can be extended to a heat releasing turbulent flow, allowing an estimate to the approximation given by the series solution for the temperature distribution.

1. Formulation of Equations

For the case of fully developed turbulent flow in a pipe of radius \( a \) defined by

\[
0 \leq \eta = \frac{r}{a} \leq 1, \quad 0 \leq \theta \leq 2\pi, \quad -\infty \leq \xi = \frac{x}{a} \leq +\infty
\]

the average velocity components and fluctuations are assumed

\[
\overline{u} = 0, \quad \overline{v} = 0, \quad \overline{w} = \overline{w}(\eta), \quad u' = u, \quad v' = v, \quad w' = w - \overline{w},
\]

along the cylindrical coordinates \( \eta, \theta, \xi \) respectively.

The absolute temperature at any point has a time average \( \overline{T} \) and a fluctuation \( T' \) of zero mean value. The mean value of the eddy heat transfer fluctuation \( \phi u' T' \) is assumed to be a function of \( \eta \) only. According to the 'momentum transfer theory' [2, 4, 5, 6]

\[
-\overline{u}' \overline{T}'(\eta) = \frac{1}{a} \varepsilon_H \frac{\partial \overline{T}}{\partial \eta},
\]

where \( \varepsilon_H = \alpha \varepsilon_M \) is the eddy diffusivity for heat and

\[
\varepsilon_M(\eta) = \frac{-v f_2(\eta)}{a f_1(\eta)/d \eta}
\]

defines the eddy diffusivity for momentum. The ratio \( \alpha \) will be assumed \( \alpha = 1 \) [4, 5, 6], \( v \) is the kinematical coefficient of viscosity and \( f_1(\eta), f_2(\eta) \) are the Pai polynomials expressing the time average axial velocity distribution \( \overline{w}(\eta) = \overline{w}(0) f_1(\eta) \) and correlation of velocities

\[
\overline{u}' \overline{w}'(\eta) = \left( \frac{v}{a} \right) \overline{w}(0) f_2(\eta)
\]

respectively. With the usual designation \( Pe \) is the Péclet number \( Pe = Pr Re \), defined as product of the Prandtl number \( Pr = \mu c/k \) and Reynolds number \( Re = 2 a \overline{w}_m/v \), where \( c \) is the specific heat, \( k \) the thermal conductivity and \( \overline{w}_m = \overline{w}(0) \) the maximum of \( \overline{w}(\eta) \) at the pipe axis \( \eta = 0 \).

It will be considered that the fluid and wall of the pipe are maintained at a constant temperature \( \overline{T}(0, \eta) \) up to a certain cross section \( \xi = 0 \) at which the temperature of the fluid at the wall is changed to a different temperature.