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REFERENCES


Zusammenfassung

Die Arbeit befasst sich mit der exakten Lösung des bekannten Ausflussproblems für ideale Flüssigkeiten. Die abgeleiteten Ausdrücke sind gültig für Gefäße von konstantem Querschnitt. Die exakte Lösung für die Ausflusszeit wird mit dem Fall vernachlässigbar kleiner Trägheitshöhe verglichen. Wo eine exakte Lösung in geschlossener Form unmöglich erschien, wurde ein Analogrechner benutzt.

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The Influence of Finite Electrical Conductivity
on the Development of a Magnetically Driven Shock
in MGD-(β ≪ 1)-Approximation

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1. Introduction

The mathematical treatment of the MGD basic equations is rendered extremely difficult by dissipation terms, as, e.g., caused by thermal conductivity, internal friction or finite electrical conductivity of the medium. Such terms give rise to a parabolic degeneration of the otherwise hyperbolic equations [1, 2, 3] and, consequently, exclude the application of the method of characteristics which is usually used in the non-dissipative case [3, 4, 5]. In Chapter 2, a numerical method of solution for the parabolically degenerated one-dimensional MGD basic equations is given; it has been developed in analogy to the theory of characteristics. This method is advantageous

1) The research described in this document was made possible by a research grant from the US government.
2) Numbers in brackets refer to References, p. 91.
insofar as it has—particularly for weak dissipation—similar stability properties as
the method of characteristics (in the case of vanishing dissipation). Using this
numerical method, in Chapter 4 the unsteady development of the structure of a
magnetically driven shock under the influence of finite electrical conductivity is
investigated in MGD-($\beta \ll 1$)-approximation. The ($\beta \ll 1$)-approximation was chosen
since it leads to a particularly simple and perspicuous form of the basic equations.

2. Method of Solution

The one-dimensional MGD basic equations are given in the form:

$$u_t + A u_x = R u_{xx} = \varphi .$$  \hfill (1)

The components of the column vector $u$ are the dependent variables. $R$ is a constant
diagonal matrix and determines the degree of dissipation. We suppose that the
matrix $A = a_{ik}$ ($i, k = 1, 2 \ldots n$) has $n$ real eigenvalues $\lambda^{(i)}$ defining $n$ linearly inde-
dependent left-hand eigenvectors $l^{(i)}$ as solutions of the equations

$$l^{(i)} (A - \lambda^{(i)} I) = 0 .$$  \hfill (2)

If initially the dissipation term $\varphi$ is considered an inhomogeneity, (1) can be put into
the following „characteristic“ form:

$$l^{(i)} (u_t + A u_x) = l^{(i)} \varphi .$$  \hfill (3)

i.e.,

$$l^{(i)} \frac{\partial u}{\partial s^{(i)}} = l^{(i)} \varphi \quad \hfill (4)$$

with

$$\frac{\partial}{\partial s^{(i)}} = \frac{\partial}{\partial t} + \lambda^{(i)} \frac{\partial}{\partial x} .$$  \hfill (5)

In order to evaluate (4) numerically, $\varphi$ is approximated as a difference quotient, e.g.,
in the form

$$\varphi = R \frac{(u_{m-1,n} - 2 u_{m,n} + u_{m+1,n})}{(\Delta x)^2} .$$  \hfill (6)

It is assumed that this formulation of $\varphi$ as well as the numerical evaluation of the
left-hand side of (4) is based on a rectangular lattice $(m, n)$ (Figure 1) with the lattice
distances $\Delta x$ and $\Delta t$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{difference_scheme.png}
\caption{Difference scheme.}
\end{figure}