Nonlinear waves have been studied in a circular waveguide partially filled with a cold magnetized collisionless plasma. The solution obtained is periodical with the frequency increasing as a square root of the amplitude of the solution in the first approximation.

1. INTRODUCTION

Propagation of waves of finite amplitude in cold collisionless, homogeneous plasma is already well known: A case of waves propagating along a magnetostatic field is solved in [1] and a special case of a soliton is discussed in [2]. Propagation of waves of finite amplitude across the magnetostatic field has been studied in [3] and the case of a soliton in [4]. However, comparatively little is known as yet about an effect of inhomogeneities and boundaries of a medium on the propagation of nonlinear waves. Electrostatic waves propagating parallel to a strong magnetostatic field through a waveguide completely filled with homogeneous plasma were studied in [5] and [6], both theoretically and experimentally. Nonlinear waves in a pinch discharge are treated in [7].

In the present paper we will be interested in the propagation of waves of finite amplitude through a circular waveguide partially filled with a cylinder of homogeneous two-component plasma of a smaller radius than that of the waveguide. We consider a collisionless cold plasma, the magnetostatic field being homogeneous and parallel to the waveguide axis. We confine ourselves only to azimuthally symmetric processes, stationary in the coordinate system moving with the wave.

2. BASIC EQUATIONS

Let us consider a cylinder of a homogeneous plasma with the radius $h^*$, surrounded by a conducting cylinder of the radius $R^* > h^*$. Introduce the cylindrical coordinate system ($r^*$, $\varphi^*$, $z^*$) in which the $z^*$-axis is identical with the waveguide axis. The motion of the plasma will be described, in accordance with the assumptions, by the
following equations:

\[
\begin{align*}
\frac{\partial n^*}{\partial t^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* n^* v_{r^*}^*\right) + \frac{\partial}{\partial z^*} \left(n^* v_z^*\right) &= 0, \\
\frac{\partial n^*}{\partial t^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* n^* V_{r^*}\right) + \frac{\partial}{\partial z^*} \left(n^* V_z^*\right) &= 0, \\
d^* v^* &= -\frac{e}{m} E^* - \frac{e}{mc} \left[v^* \times B^*\right], \\
D^* v^* &= \frac{e}{m} E^* + \frac{e}{Mc} \left[v^* \times B^*\right], \\
\nabla \times E^* + \frac{1}{c} \frac{\partial B^*}{\partial t^*} &= 0, \\
\nabla \times B^* - \frac{1}{c} \frac{\partial E^*}{\partial t^*} &= \frac{4\pi}{c} en^* (v^* - v^*), \\
\nabla \cdot B^* &= 0.
\end{align*}
\]

Here, \(n^*\) is a density, \(m, M, v^*, V^*\) are masses and velocities of electrons and ions respectively, \(E^*, B^*\) are electric and magnetic field vectors resp., \(d^* = \partial/\partial t^* + v_r^* \partial/\partial r^* + v_z^* \partial/\partial z^*\), \(D^* = \partial/\partial t^* + V_r^* \partial/\partial r^* + V_z^* \partial/\partial z^*\). In the coordinate system \((r^*, \varphi^*, z^*)\) moving with the wave with the velocity \(v_0^* \ll c\) it is \(z_1^* = z^* - v_0 t^*\). Introduce dimensionless variables and change the scale for some of them by introducing a small parameter \(\varepsilon\):

\[
\begin{align*}
\frac{r}{r^*} &= h^*/h^*, & z = \varepsilon z_1^*/h^*, & v_r = \varepsilon^{-1} v_{r_0}^*/v^*_0, & v_{\varphi} = v_{\varphi}/v^*_0, \\
v_z = v_z^*/v^*_0, & V_r = \varepsilon^{-1} V_{r_0}^*/v^*_0, & V_{\varphi} = V_{\varphi^*}/v^*_0, & V_z = V_z^*/v^*_0, \\
B_r = \varepsilon^{-1} B_{r_0}^*/B_0^*, & B_{\varphi} = B_{\varphi^*}/B_0^*, & B_z = B_z^*/B_0^*, & n = n^*/n_0^*,
\end{align*}
\]

where \(n_0^*\) is unperturbed density, \(B_0^*\) is the magnitude of unperturbed magnetostatic field. The change of the scale of some variables means an assumption that the radial components of velocities and of magnetic field are less than remaining components and that \(\partial/\partial z^* \ll \partial/\partial r^*\). We introduce further dimensionless constants \(\mu = M/m, A = v_0^*(h^*/\omega_{ce})\), \(K = A^{-1}(v_0^*)^2 4\pi n_0^* m (B_0^*)^2\), where \(\omega_{ce}\) is the electron cyclotron frequency.

3. FIRST APPROXIMATION

From equations (1) to (7) we exclude the electric field (for the sake of brevity, we shall not also give the expressions for the electric field in the results) and the remaining equations are solved by means of successive approximations (see e.g. [7]) when the