Zusammenfassung

Es wird die Strömung einer viskoelastischen, elektrisch leitenden Flüssigkeit untersucht auf Grund der elektromagnetischen Grundgleichung und der für solche Flüssigkeiten geltenden Bewegungsgleichungen. Im besonderen wird die Strömung zwischen parallelen Platten erörtert. Sie ist charakterisiert durch zwei dimensionslose Kennzahlen, die Hartmannsche Zahl und die Elastizitätszahl. Das elastische Verhalten bewirkt eine Verflachung des Geschwindigkeitsprofils und verändert auch den Druck und die Schubspannungen.

(Received: March 24, 1961.)

Unsteady Low Prandtl Number Heat Transfer for Flow Across a Flat Plate

By Robert D. Cess, Raleigh, N.C., U.S.A.

1. Introduction

Heat transfer to laminar flow across a flat plate with an unsteady surface temperature has recently been considered in Reference 1\(^1\), and an approximate solution was obtained for the case of a step change in surface temperature with time. In order to gain a further understanding of this type of unsteady convection process, it is the purpose of the present note to investigate an alternate method of analyzing the problem. In particular, consideration will be given to fluids having very low Prandtl numbers (liquid metals).

The physical model and coordinate system are illustrated in Figure 1, and laminar flow of a constant-property fluid is assumed. Initially the plate surface is at the free-stream temperature \( T_\infty \), whereas at \( t = 0 \) the plate surface is changed to and maintained at a constant temperature \( T_w \).

\[
\begin{align*}
T &= T_\infty, \quad t < 0 \\
T &= T_w, \quad t > 0
\end{align*}
\]

Figure 1

Physical model and coordinate system.

\(^1\) This research was supported by the National Science Foundation through Grant G-1487 5.
\(^2\) Mechanical Engineering Department, North Carolina State College. Present address: College of Engineering, State University of New York, Long Island, New York.
\(^3\) Numbers in brackets refer to References, page 165.
2. Analysis

Under the conditions of the present problem, the energy equation becomes

\[ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} \]  

(1)

where \( \alpha \) is the thermal diffusivity of the fluid, and \( \theta \) is defined as

\[ \theta = \frac{T - T_\infty}{T_w - T_\infty}. \]

It readily follows that the boundary conditions are

\[ \theta = 0; \quad x = 0, \quad \theta = 0; \quad t = 0, \]
\[ \theta \rightarrow 0; \quad y \rightarrow 0, \quad \theta = 1; \quad y = 0, \quad x > 0, \quad t > 0. \]

Furthermore, the velocity components \( u \) and \( v \) occurring within Equation (1) are given by the well-known Blasius relations [2]

\[ u = u_\infty f', \quad v = \frac{1}{2} \sqrt{\frac{v u_\infty}{x}} (\eta f' - f) \]  

(2)

where \( v \) denotes the kinematic viscosity of the fluid and

\[ \eta = y \sqrt{\frac{u_\infty}{v x}}. \]

It is now desired to introduce an approximation regarding the velocity components which will be appropriate for very low Prandtl numbers. With this in mind, consider first steady-state heat transfer. From Reference [3] a suitable approximation for \( f(\eta) \) applicable to low Prandtl number fluids is the large-\( \eta \) expression \( f = \eta - \beta \), where \( \beta = 1.721 \). In turn, Equations (2) yield

\[ u = u_\infty, \quad v = \frac{\beta}{2} \sqrt{\frac{v u_\infty}{x}} \]  

(3)

which, of course, represent the flow occurring outside the velocity boundary layer. As discussed in Reference [3], the physical reasoning behind this assumption is that the steady-state thermal boundary layer is much thicker than the velocity boundary layer. Correspondingly, throughout the major portion of the thermal boundary layer the flow field is described by Equations (3).

Consider next the unsteady case. In this situation the thermal boundary layer thickness is initially zero and increases with time to its eventual steady-state value. This implies that the conditions of the foregoing assumption would not be satisfied during some initial interval of time. Nevertheless, during the initial stages of the heat transfer process the influence of convection will be small\(^4\), such that any error in the formulation of the velocities should not appreciably affect the overall results. Equations (3) would therefore appear applicable to the present investigation providing the Prandtl number is sufficiently small.

So, utilizing Equations (3) in Equation (1) and defining two new independent variables as

\[ \tau = \frac{u_\infty t}{x}, \quad \xi = \frac{y}{\sqrt{v t}} = \frac{\eta}{\sqrt{x}} \]

\(^4\) For example, see Reference [1].