Electron Optics in Multi-Stage Lens System I

By Minoru Morikawa, Kyoto, Japan

1. Introduction

In the high voltage Cockcroft-Walton accelerators\(^2\), the optics of the accelerating tubes can be simplified by treating them as one thick lens, especially when the lenses are numerous.

When the tubes are composed of a series of the same form electrodes and the potential differences between them are same, the simple treatment is possible.

In the preceding paper\(^5\), the expressions for the cardinal elements of such lens system as a whole, which is composed of the two-cylinder lenses with infinitesimally small gaps, are given.

We extended here, these expressions of the over all cardinal elements of the lens system, for the case that each two-cylinder lens has the finite gap length between the cylinders.

2. General Properties of Electron Lens

The path of the charged particle in the axis-symmetrical electrostatic field is considered. By introducing \(R = r \Phi^{1/4}\), the path equation, in the paraxial ray approximation, is given by

\[ R''(Z) + T(Z) R(Z) = 0, \]

where

\[ T(Z) = \frac{3}{16} \left( \frac{\Phi'}{\Phi} \right)^2. \]

To calculate the cardinal elements of the two-cylinder lens, two special solutions of Equation (1) are considered: one corresponding to the path coming from the left parallel to the axis with \(R = 1\), before passing through the lens field, and the other to the path coming from the right parallel to the axis with \(R = 1\). By denoting these two solutions by \(R^{(1)}\) and \(R^{(2)}\),

\[ R^{(1)} \rightarrow 1 \text{ for } Z \rightarrow + \infty \text{ and } R^{(2)} \rightarrow 1 \text{ for } Z \rightarrow - \infty. \]

The paths, after passing through the lens field, are expressed asymptotically

\[ R^{(1)} \rightarrow A_1 + \frac{Z}{j} \text{ for } Z \rightarrow - \infty \text{ and } R^{(2)} \rightarrow A_2 - \frac{Z}{j} \text{ for } Z \rightarrow + \infty, \]

since \(R^{(1)}(- \infty) = - R^{(2)}(+ \infty)\) can be shown by using that the Wronskian is constant.

From these, the cardinal elements of the lens are given by

\[ f_1 = \left( \frac{\Phi_1}{\Phi_2} \right)^{1/4}/, \quad f_2 = \left( \frac{\Phi_2}{\Phi_1} \right)^{1/4}/, \quad Z(F_1) = -/A_1, \quad Z(F_2) = /A_2. \]

\(^1\) Engineering Department, Nissin Electric Co., Ltd., Kyoto, Japan.
\(^2\) For the accelerating tubes of the multi-stage lens system, see, for example, E. B. Bas, L. Preuss, and W. Schneider, ZAMP 16, 583 (1959); G. Henneberke, Nucl. Instr. & Meth. 7, 89 (1960).
\(^5\) M. Morikawa, J. appl. Phys. (to be published).
The asymptotic form of $R$ can be calculated, by the perturbation method, as follows. We expand $R$ and $T$ in Equation (1) in powers of the perturbation parameter $1/\gamma$, where $\gamma$ is defined by

$$\gamma = \frac{\Phi_2 + \Phi_1}{\Phi_2 - \Phi_1}. \quad (6)$$

For the paths $R^{(1)}$ and $R^{(2)}$, these expansions are

$$R = 1 + \frac{1}{\gamma^2} R_2 + \frac{1}{\gamma^3} R_3 + \cdots, \quad T = \frac{1}{\gamma^2} T_2 + \frac{1}{\gamma^3} T_3 + \cdots. \quad (7)$$

By these, we can obtain the equations that represent various orders of the perturbation, i.e.,

$$R_{n}^{*} = T_{n}, \quad R_{n}^{*} = T_{n} + T_{n} R_{2}, \quad \text{etc.} \quad (8)$$

By solving these, for the given potential $\Phi$ on the axis, we can obtain the asymptotic form of $R^{(1)}$ and $R^{(2)}$.

3. Cardinal Elements of Two-Cylinder Lens

In the equi-diameter two-cylinder lens, the diameter $D$, and the accelerating potential $V$, are taken as a unit of length and a unit of potential, respectively.

At first, the two-cylinder lens with infinitesimally small gap between the cylinders are considered. The potential on the axis of this lens is given by

$$\Phi = \frac{1}{2} (\gamma + \tanh \omega Z), \quad (9)$$

where $\gamma = (\Phi_2 + \Phi_1)/(\Phi_2 - \Phi_1)$, and $\omega = 2 \cdot 636$. The cardinal elements of this lens are given, by the perturbation method with $1/\gamma$:

$$\frac{1}{f} = \frac{\omega}{4 \gamma^2} \left[ 1 + \frac{25}{48 \gamma^2} + O\left( \frac{1}{\gamma^4} \right) \right],$$

$$Z(F_1) = -4 \frac{\gamma^2}{\omega} \left[ 1 - \frac{25}{48 \gamma^2} + \frac{1}{8 \gamma^3} + O\left( \frac{1}{\gamma^4} \right) \right],$$

$$Z(F_2) = 4 \frac{\gamma^2}{\omega} \left[ 1 - \frac{25}{48 \gamma^2} - \frac{1}{8 \gamma^3} + O\left( \frac{1}{\gamma^4} \right) \right]. \quad (10)$$

Next, we consider the two-cylinder lens with the gap length $2a$. For this, the potential on the axis is approximated by

$$\Phi = \frac{1}{2} \left( \gamma + \frac{1}{2 \omega a} \log \left[ \frac{\cosh \omega (Z + a)}{\cosh \omega (Z - a)} \right] \right), \quad (11)$$

where the potential variation at the gap is assumed to be linear.

By the perturbation method, as above, the cardinal elements of this are given by

$$\frac{1}{f} = \frac{\omega(a)}{4 \gamma^2} \left[ 1 + \frac{25}{48 \gamma^2} g_1(a) + O\left( \frac{1}{\gamma^4} \right) \right],$$

$$Z(F_1) = -4 \frac{\gamma^2}{\omega(a)} \left[ 1 - \frac{25}{48 \gamma^2} g_1(a) + \frac{1}{8 \gamma^3} g_2(a) + O\left( \frac{1}{\gamma^4} \right) \right],$$

$$Z(F_2) = 4 \frac{\gamma^2}{\omega(a)} \left[ 1 - \frac{25}{48 \gamma^2} g_1(a) - \frac{1}{8 \gamma^3} g_2(a) + O\left( \frac{1}{\gamma^4} \right) \right]. \quad (12)$$