An Induction Motor Model with Deep-Bar Effect and Leakage Inductance Saturation

G. J. Rogers and D. S. Benaragama, Southampton

Contents: A two-phase dynamic induction motor model is derived which includes the effects of rotor bar eddy currents and leakage inductance saturation. The model is based on approximate analysis of the air-gap and slot electromagnetic fields which leads directly to lumped circuit equations. New expressions are given for the variation of the zig-zag and slot leakage inductance with saturation. — The model is applied to the calculation of both steady state and dynamic characteristics for a large induction motor.

Ein Modell des Asynchronmotors mit Hochstabläufereffekt und Sättigung der Streuinduktivität


List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>A</td>
<td>magnetic vector potential</td>
</tr>
<tr>
<td>B</td>
<td>flux density</td>
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<tr>
<td>G</td>
<td>describing function</td>
</tr>
<tr>
<td>g</td>
<td>air-gap length</td>
</tr>
<tr>
<td>H</td>
<td>magnetic field intensity</td>
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<tr>
<td>(I_{0m})</td>
<td>rotor saturation current</td>
</tr>
<tr>
<td>(I_r)</td>
<td>rotor current matrix</td>
</tr>
<tr>
<td>(I_{j})</td>
<td>current in the jth rotor slot</td>
</tr>
<tr>
<td>(I_{mn})</td>
<td>amplitude of the fundamental of (I_{j}) distribution</td>
</tr>
<tr>
<td>(I_{st})</td>
<td>rotor harmonic conceptual current matrix</td>
</tr>
<tr>
<td>(I_s)</td>
<td>stator current matrix</td>
</tr>
<tr>
<td>(I_{st})</td>
<td>stator harmonic conceptual current matrix</td>
</tr>
<tr>
<td>(i_{m})</td>
<td>(m)th modal eddy current in the (j)th rotor slot</td>
</tr>
<tr>
<td>(i_{ab})</td>
<td>(m)th harmonic conceptual current at the interface between (j)th rotor slot and slot neck regions</td>
</tr>
<tr>
<td>(i_{se})</td>
<td>(m)th harmonic conceptual current at the opening of the (j)th rotor slot</td>
</tr>
<tr>
<td>(i_{0b})</td>
<td>rotor zero order conceptual current matrix</td>
</tr>
<tr>
<td>(i_{0l})</td>
<td>conceptual current at the (j)th rotor slot due to saturation of the tooth tips</td>
</tr>
<tr>
<td>(K_{mn})</td>
<td>(m)th harmonic winding distribution factor</td>
</tr>
<tr>
<td>(l)</td>
<td>effective length of the motor</td>
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<tr>
<td>(L_{nc})</td>
<td>self inductance of the eddy current mode (i_{0m})</td>
</tr>
<tr>
<td>(L_r)</td>
<td>rotor inductance matrix</td>
</tr>
<tr>
<td>(L_s)</td>
<td>stator inductance matrix</td>
</tr>
<tr>
<td>(p)</td>
<td>pole pitch</td>
</tr>
<tr>
<td>(M_{n0})</td>
<td>mutual inductance between (i_{0m}) and (I_{j})</td>
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<tr>
<td>(M_{sl})</td>
<td>mutual inductance matrix between rotor winding and rotor modal eddy currents</td>
</tr>
<tr>
<td>(M_{ir})</td>
<td>mutual inductance matrix between rotor winding and rotor conceptual winding</td>
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<tr>
<td>(M_{ir})</td>
<td>mutual inductance matrix between rotor winding and rotor conceptual winding</td>
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<tr>
<td>(M_{is})</td>
<td>mutual inductance matrix between rotor winding and stator conceptual winding</td>
</tr>
<tr>
<td>(M_{isbn})</td>
<td>mutual inductance matrix between stator winding and rotor conceptual winding</td>
</tr>
<tr>
<td>(M_{sns})</td>
<td>mutual inductance matrix between stator winding and stator conceptual winding</td>
</tr>
<tr>
<td>(N_{1}, N_{2})</td>
<td>number of stator and rotor slots</td>
</tr>
<tr>
<td>(R_{e})</td>
<td>resistance of eddy current mode (i_{0m})</td>
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<tr>
<td>(R_s)</td>
<td>rotor resistance matrix</td>
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<tr>
<td>(R_{st})</td>
<td>stator resistance matrix</td>
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<tr>
<td>(s_1, d_1)</td>
<td>dimensions of a stator slot</td>
</tr>
<tr>
<td>(s_2, d_2)</td>
<td>rotor slot opening and rotor slot neck height</td>
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<tr>
<td>(s_0, d_0)</td>
<td>dimensions of a rotor bar</td>
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<tr>
<td>(s_p)</td>
<td>slot pitch</td>
</tr>
<tr>
<td>(N_{c})</td>
<td>number of turns per slot</td>
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<tr>
<td>(U_0)</td>
<td>applied voltage matrix</td>
</tr>
<tr>
<td>(q)</td>
<td>number of slots per pole phase group</td>
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<td>(\psi_r)</td>
<td>rotor flux linkage matrix</td>
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</table>
1 Introduction

The terminal characteristics of induction motors are critically dependent on rotor resistance and the total leakage reactance. Generally neither parameter remains constant. Rotor resistance varies due to eddy currents which flow in the solid cage rotor conductors. Leakage reactance is also varied by the rotor bar eddy currents but a more significant change is caused by saturation of the iron associated with the leakage flux paths in the machine. Both effects have been examined by detailed analysis of a single slot under steady state conditions. Analytical and numerical methods have been used to study the problem of deep-bar effect. [1–4]. Leakage saturation is less amenable to analysis but there are two notable approximate treatments of the phenomenon. Agarwal and Alger [5] treat each slot in isolation and one intuitively feels that the saturation in practice would differ with the position of the conductor in the air-gap. Chalmers and Dodgson [6] have recognised this by introducing a saturation factor which allows for the proportion of slots carrying currents below the level at which saturation occurs. It is difficult to extend their detailed steady state, zero speed study to the dynamic regime.

In this paper we develop an induction motor model which includes the effects or rotor bar eddy currents and leakage saturation. The model is an extension of that recently derived by the author [7] using a piece-by-piece method [8] to analyse the air-gap field. In the model, eddy currents are represented by extra short-circuited coils coupled to the main rotor circuit. The resistances and inductances of these equivalent eddy current circuits are constant and do not vary with speed or supply frequency but depend solely on the slot dimensions. Each slot is initially considered in isolation but in assessing the overall machine performance its position and the state of the local field is taken into account.

2 The Model Geometry

A developed model of a two-pole, three-phase induction motor is shown in Fig. 1. On the stator there is a three phase, single layer, full pitched winding in open slots. On the rotor there is a cage winding in semi-closed slots. The iron is considered infinitely permeable except for the rotor tooth tip region in which we assume the ideal magnetisation characteristic shown in Fig. 2.

Following the author’s earlier paper the air-gap region, the stator slot region and the rotor slot region are considered separately. The similarity between the slot geometries allows attention to be limited to one typical stator slot and one typical rotor slot. These regions are shown in Fig. 3. The rotor slot region is more detailed than that previously studied by this method. It can be seen to include the saturable tooth tip. Analysis of the stator slot region has already been presented. In the air-gap field extra terms are introduced by tooth tip saturation.

The induction motor model normally used for dynamic analysis is obtained by neglecting all but the fundamental interaction between stator and rotor fields. In order to simplify the form of the final model we adopt the same policy in this paper.