BOUNDARY CONDITIONS, SYMMETRY BREAKDOWN
AND FEYNMAN PATH INTEGRAL*)

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The analysis of boundary conditions in variational formulation of classical field theory is
performed. It is used in the Feynman path integral formulation of quantum theory including non-
trivial cases of dynamical symmetry breakdown.

It is widely known that in conventional handling a Feynman path integral (FPI)
we cannot reach physically interesting theories such as spontaneously broken sym-
metry solutions. There are attempts to formulate quantization procedures around
classical solutions [1], but they are mostly semiclassical and do not cover dynamical
symmetry breakdown. In this note I want to show a possible way how one can ap-
proach the correct vacuum in closed systems (where it is translationally invariant)
and to hint to the connection with boundary conditions of classical fields. Origin
of the problems with symmetry breaking solutions lies in classical physics. When
we want to include such solutions, we must deal with infinite energies. Then the ques-
tion arises what solutions are the correct ones, what is their asymptotic behaviour and
what surface terms really disappear. We must analyse the solutions and from a phy-
sical principle we decide for a correct class of solutions. Then we renormalize energy
and other possible infinities to finite quantities. The analysis of classical solutions
will be done with respect to conserved charges (especially energy) which leads to the
analysis of boundary conditions or initial value data for these solutions. For this
purpose I shall use the variational formulation of classical dynamical problem and
FPI for quantum problem since they have many common features. From the analysis
it will be seen how to formulate correctly theories with possible broken symmetry
solutions. Using conserved Noether charges we divide boundary conditions (solu-
tions) into classes in such a way that solutions from different classes differ in infinite
values of energy or other possible charges. We can parametrize these classes by a con-
stant and this constant is a new implicit parameter in the theory. From these classes
we select the physical ones by the claim of stability of the theory that reads: there exists
a translationally invariant solution minimizing the energy. With respect to such a so-
lution we renormalize energy and other quantities to have finite relative values. Such
minimal energy solution we shall call classical (quantum) vacuum.

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A. Classical theory

In classical physics we have a problem of finding stationary trajectories of the action on a region $\Omega \subset R^4$. Let us consider

$$\mathcal{A}(\phi) = \int_{\Omega} d^4x \left[ \frac{1}{2} (\partial_{\mu} \phi)^2 - V(\phi) \right] = \int_{\Omega} d^4x \mathcal{L}(\phi).$$

Whenever $\int_{\Sigma} d\sigma^\mu \frac{\delta \mathcal{L}}{\delta \partial_{\mu} \phi} \delta \phi = 0$, the stationary trajectories fulfil the Euler-Lagrange equation [2]:

$$\partial^2 \phi(x) + V'(\phi(x)) = 0, \quad V' \equiv \frac{\delta V}{\delta \phi}.$$ 

We shall specify the region $\Omega$ to consist of two spacelike hypersurfaces $\Sigma_+, \Sigma_-$ at times $T, -T$, respectively, for a particular observer, and a cylinder $K$ joining these two hypersurfaces. We shall perform a limit with $K$ to spatial infinity. We specify the following boundary conditions for (1) or (2):

$$r \to \infty, \quad r$$ is spatial radial coordinate

and

$$\phi|_{\Sigma_-} = \phi_-, \quad \phi|_{\Sigma_+} = \phi_+$$

or

$$\phi|_{\Sigma_-} = \phi_-, \quad \phi|_{\Sigma_-} = \phi_-.$$

With such boundary conditions the conservation laws are valid, i.e.

$$Q(\Sigma_+) = Q(\Sigma_-).$$

But these charges can be infinite (due to infinite volume of $\Sigma$'s) and the conserved charges are not well defined. As we are interested rather in relative charges of two solutions $\phi_1, \phi_2$, instead in their absolute values, we define the relative charge

$$AQ(\phi_1, \phi_2) = \int_{\Sigma} d^3x \left[ j_0(\phi_1) - j_0(\phi_2) \right],$$

where the difference is taken before limiting with $K$ to infinity. With respect to such a relative charge we divide the boundary conditions into classes of finite relative charges. Then we can restrict ourselves to one of these classes. The suitable charge for this purpose is energy

$$E(\phi) = \int_{\Sigma} d^3x \ T_{00}(\phi) = \int_{\Sigma} d^3x \left\{ \frac{1}{2} [\phi^2 + (\partial_k \phi)^2] + V(\phi) \right\}.$$