UNSTABLE SYSTEMS AND REPEATED MEASUREMENTS

III. EXAMPLE (HOMOGENEOUS CHAMBER), CONJECTURE FOR THE GENERAL CASE AND DISCUSSION

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In the last part of this paper we treat the example concerning the behaviour of an unstable system in a homogeneous measuring device. This case was studied by FONDA et al. (N. Cim. 15 A (1973), 689); we obtain similar results, however, some comments have to be made on their paper. Further we make a conjecture concerning the measured decay law in the general case. Finally, we discuss in which way the results could be used to search for non-exponentialities of the primary decay law.

The first two parts of the paper (referred to hereafter as [I], [II]) contain formulation of the problem and two examples. We shall start this part by the third example.

Section 4 (continued)

Example 3 (homogeneous chamber): Let us assume that the function $\phi(t)$ is a constant, $\phi(t) = \lambda$ for all $t$, i.e. $W(t) = \exp (-\lambda t)$. This assumption corresponds to a special case of periodically structured chamber (any positive number may play the role of period $t_c$). The homogeneous chamber represents itself a good model for the real bubble chambers. Let us notice that the present case was investigated by FONDA et al. (see especially Ref. [6]); our results are similar to theirs, though their basic equation is otherwise formulated (see remark at the end of this example).

The basic equation (2.4) is now of the form

$$f(t) = P(t) + \lambda \int_0^t d\xi P(t - \xi) f(\xi).$$

This equation can be conveniently solved (as any convolution-type equation) by means of the Laplace transformation. We shall denote the Laplace transforms as follows:

$$\mathcal{L}(P)(s) = \int_0^\infty d\tau e^{-\tau s} P(\tau), \quad \mathcal{L}(f)(s) = \int_0^\infty d\tau e^{-\tau s} f(\tau), \quad \mathcal{L}(t^k P)(s) = \int_0^\infty ds e^{-\tau s} t^k P(\tau) \quad \text{etc.}$$

Properties of the solution are the following:

**Lemma:** Let the assumptions (D1), (D2) be valid, and let the inequality $\nu_o T = \lambda T > 1$ hold. Then the solution of Eq. (4.16) is of the form

$$f(t) = (A + B(t)) \exp (\nu_o t),$$

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where \( v_0 > 0, A > 0, B(t) \) is a continuous function, and there exist \( v_1, 0 \leq v_1 < v_0 \), such that for any \( \alpha < v_0 - v_1 \) it holds

\[
e^{\alpha t} B(t) \in L^2(0, \infty).
\]

Here

\[
A = \left[ \lambda^2(\tilde{tP})(v_0) \right]^{-1}
\]

and \( v_0 \) is the real and positive solution of the equation

\[
\lambda \int_0^\infty P(t) e^{-\gamma t} dt = 1;
\]

under the considered assumptions it exists and is given uniquely. Further, if other solutions of Eq. (4.17) exist, then \( v_1 \) is the maximum of their real parts, otherwise \( v_1 = 0 \). Proof of this lemma is contained in Appendix E.

The proved lemma implies easily the following result:

**Theorem 3:** Let (i) the chamber be homogeneous \( \varphi(t) = \lambda = v_0 \),

(ii) the assumptions of the preceding lemma be valid.

Then the non-decay probabilities are of the form

\[
E_i(t) = [A_i + B_i(t)] \exp (-\gamma t), \quad i = I, II,
\]

where

\[
A_I = A, \quad A_{II} = \frac{\lambda A}{\lambda - \gamma};
\]

for any \( \alpha < v_0 - v_1 \) the following relations hold

\[
e^{\alpha t} B_I(t) \in L^2(0, \infty),
\]

\[
\lim_{t \to \infty} e^{\alpha t} B_{II}(t) = 0. \quad (4.18c)
\]

Here \( \gamma, 0 < \gamma < \lambda \), is the unique solution of the equation

\[
\lambda \int_0^\infty P(t) e^{(\gamma - \lambda)t} dt = 1. \quad (4.17a)
\]

**Proof:** Let us define \( \gamma = \lambda - v_0 \). The inequality (2.6) implies \( v_0 < \lambda \), and consequently \( 0 < \gamma < \lambda \). The results concerning \( E_i(t) \) follow directly from the lemma.

\[^{1)}\] Notice that the condition (4.18b) is weaker than (4.18c), even if they could coincide in particular models. The non-decay probability \( E_{II}(t) \), and consequently the function \( B_{II}(t) \), is, however, the more interesting one from the physical point of view.