PHOTON COUNTING STATISTICS OF SUPERPOSITION OF ONE-MODE COHERENT LIGHT AND CHAOTIC LIGHT CONSISTING OF TWO LORENTZIAN SPECTRAL LINES

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The approximate photon counting distribution is calculated for the polarized superposition of one-mode coherent light and chaotic light consisting of two Lorentzian spectral lines. Further the exact and approximate third factorial moments of the photon counting distribution are proposed in this case. Also the second factorial moment for the superposition of one-mode coherent light and chaotic light consisting of \( n \) Lorentzian spectral lines is given.

1. INTRODUCTION

The photon counting statistics of the polarized chaotic light consisting of two Lorentzian spectral lines has been studied in [1,2]. The generating function for the photon counting distribution in the case that the Lorentzian lines have the same central frequency but different halfwidths has been considered in [1], whereas the case when central frequencies of the Lorentzian lines are different but the halfwidths are the same has been studied in [2]. Also the first moments of the photon counting distribution for the general case when the Lorentzian lines have different central frequencies and different halfwidths have been given in [1].

In this paper we study the photon counting statistics of the polarized superposition of one-mode coherent light and chaotic light consisting of two Lorentzian spectral lines. We calculate the second factorial moment of the photon counting distribution for the superposition of one-mode coherent light and chaotic light consisting of \( n \) Lorentzian spectral lines. The method of calculation of the approximate photon counting distribution and its factorial moments proposed in [3–5] is used (Sec. 2). Also the exact third factorial moment is calculated and compared with the approximate one (Sec. 3).

2. APPROXIMATE PHOTON COUNTING DISTRIBUTION

The photon counting distribution for the \( M \)-mode superposition of coherent and chaotic light (the mean photon number of chaotic light in every mode is equal to \( \bar{n}_{ch}/M \)) was derived in [6,7]:

\[
p(n) = \frac{1}{\Gamma(n + M)} \left( 1 + \frac{M}{\bar{n}_{ch}} \right)^{-n} \left( 1 + \frac{\bar{n}_{ch}}{M} \right)^{-M} \exp \left( -\frac{\bar{n}_c M}{\bar{n}_{ch} + M} \right) L_{n-1}^{M-1} \left( -\frac{\bar{n}_c M^2}{\bar{n}_{ch}(\bar{n}_{ch} + M)} \right),
\]

where \( \bar{n}_{ch} \) and \( \bar{n}_c \) are the mean photon numbers in the chaotic and coherent light,

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respectively, \( M \) is the number of modes and \( L_n^{M-1} \) is the Laguerre polynomial. (In the paper we assume that the photoefficiency is unity. When the photoefficiency is equal to \( \eta \) then we take \( \eta \bar{n}_c \) instead of \( \bar{n}_c \) and \( \eta \bar{n}_c \) instead of \( \bar{n}_c \).)

In order to use (1) as an approximate formula for the superposition of coherent and chaotic light with an arbitrary spectrum, we adjust the parameter \( M \) in such a way that the second factorial moment of distribution (1), which is equal to [5]

\[
\langle n(n-1) \rangle = \sum_{n=0}^{\infty} \rho(n) n(n-1) = \langle W^2 \rangle = \int_0^T \langle I(t_1) I(t_2) \rangle \, dt_1 \, dt_2 = \]

\[
= (\bar{n}_c + \bar{n}_{ch})^2 + \frac{1}{M} \left[ \bar{n}_{ch}^2 + 2\bar{n}_{ch} \bar{n}_c \right],
\]

coincides with the exact expression for the second factorial moment. For the case when one-mode coherent component and the chaotic component consisting of two Lorentzian spectral lines are considered the second factorial moment is equal to [5]

\[
\langle n(n-1) \rangle = \langle W^2 \rangle = \int_0^T \langle I(t_1) I(t_2) \rangle \, dt_1 \, dt_2 = \]

\[
= (\bar{n}_c + \bar{n}_{ch})^2 + 2\bar{n}_c \bar{n}_{ch} F(\gamma_1, \Omega_1) + 2\bar{n}_c \bar{n}_{ch} F(\gamma_2, \Omega_2) +
+ 2\bar{n}_{ch1} \bar{n}_{ch2} F(\gamma_1 + \gamma_2, \omega) + \bar{n}_{ch1}^2 F(2\gamma_1, 0) + \bar{n}_{ch2}^2 F(2\gamma_2, 0),
\]

where

\[
F(\gamma, \Omega) = 2 \frac{\gamma}{\gamma^2 + \Omega^2} + \frac{e^{-\gamma(\sqrt{\Omega^2 - 2\gamma\Omega \sin \Omega} + \Omega^2 - \gamma^2)}}{(\gamma^2 + \Omega^2)^2},
\]

\( \bar{n}_{ch} \) is the mean photon number, \( \gamma_i = \Gamma_i T, \Omega_i = (\omega_i - \omega_c) T \), \( \Gamma_i \) is the halfwidth, \( \omega_i \) is the central frequency corresponding to the \( i \)-th Lorentzian line \( (i = 1, 2) \), \( \omega_c \) is the frequency of the coherent light, \( T \) is the detection time interval, \( \omega = \omega_1 - \omega_2 = \omega_{ch} \) comparing (2) and (3) we obtain for \( M \) the following expression

\[
M = \left( \bar{n}_{ch}^2 + 2\bar{n}_c \bar{n}_{ch} \right) \left( 2\bar{n}_c \bar{n}_{ch1} F(\gamma_1, \Omega_1) + 2\bar{n}_c \bar{n}_{ch2} F(\gamma_2, \Omega_2) +
+ 2\bar{n}_{ch1} \bar{n}_{ch2} F(\gamma_1 + \gamma_2, \omega) + \bar{n}_{ch1}^2 F(2\gamma_1, 0) + \bar{n}_{ch2}^2 F(2\gamma_2, 0) \right)^{-1}.
\]

Some numerical results for the second reduced factorial moment \( \langle n(n-1) \rangle : \)

\( \bar{n}^2 - 1, \bar{n} = \bar{n}_c + \bar{n}_{ch} \), which were obtained from formula (3) are shown in Figs. 1 and 2 and the numerical results for the photon counting distribution (1) where \( M \) is given by (5) are shown in Figs. 3 and 4. From Fig. 3 it is evident that for \( \gamma_1 = \gamma_2 = \gamma \) and \( \Omega_1 = \Omega_2 = \Omega \) the curves of \( \rho(n) \) are sharpening and their peaks are shifting to higher \( n \) with increasing \( \gamma \) and \( \Omega \). One observes in Fig. 4 that also \( \rho(n) \) is sharpening and its peak is shifting to higher \( n \) again with increasing \( \gamma_2 \) if \( \gamma_1 \) and \( \Omega_1 \) and \( \Omega_2 \) are fixed.