SOME STATIC PROBLEMS IN A WAVE-GUIDE OF GENERAL CROSS-SECTION

II. A PERMANENT MAGNET

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The magnetostatic field of magnetically polarized paramagnetic matter of arbitrary shape in a straight wave-guide of general cross-section is found by the Green’s functions method. Both the static and stationary formulations are investigated and shown to give identical results.

1. INTRODUCTION

In contrast to electrostatics the magnetic case can be formulated in two ways: either as a magnetostatic or as a stationary problem. This comes from the general expressions for equivalent sources:

\[ \varrho_p = - \nabla \cdot P, \quad j_p = \frac{\partial P}{\partial t}; \]

\[ \varrho_M = - \nabla \cdot M, \quad j_M = \frac{1}{\mu} \nabla \times M. \]

From these formulae we can see that in the static limit \( \frac{\partial}{\partial t} \to 0 \) there remains only one way for the electrostatic case but still two expressions for the magnetic one.

2. THE MAGNETOSTATIC FORMULATION

Now we consider that the volume \( V_0 \) is filled with a magnetically polarized matter with the permeability \( \mu = \text{const} \). The magnetization is given by

\[ M = M(r'), \quad r' \in V_0, \quad V_0 \subset V. \]

\[ M = 0, \quad r' \notin V_0. \]

The physical situation is given in fig. 1. For details and notation see [1] and the references therein.

In the magnetostatic formulation there is no difference from the electrostatic case, except for notations (compare, e.g., the first expressions in eqs. (1) and (2)). The differential equation for the magnetic field is

\[ \Delta H = - \frac{1}{\mu} \nabla \cdot M. \]
But the boundary condition now demands that the normal component should vanish on the perfectly conducting surface, i.e.

\[ H_{\parallel} |_{r = 0} = 0. \]

From the expression (see [1], eq. (17)) valid for \( r \neq r' \in V_0 \)

\[ H_{\perp}(q, \gamma) = \frac{1}{i\gamma} \text{Grad} \; H_{\parallel}(q, \gamma) \]

and (5) we have the boundary condition for the longitudinal component

\[ \partial_{r} H_{\parallel} |_{r} = 0. \]

All results can be immediately written down from the electrostatic case. E.g., the Fourier component of the Green's function is (cf. [1], eqs. (4), (7))

\[ G(q, q'; \gamma) = \sum_{N} \frac{v_{N}(q') v_{N}(q)}{\gamma^2 + \lambda_{N}^2}, \]

and the magnetostatic potential

\[ V(r) = \frac{1}{2\mu} \sum_{N} \frac{v_{N}(q)}{\lambda_{N}} \int_{V_0} dV' \exp(-\lambda_{N}|z - z'|) \times \]

\[ \times \left[ M_{\parallel}(r') \cdot \text{Grad}' v_{N}(q') + \lambda_{N} \text{sgn}(z - z') M_{z}(r') v_{N}(q') \right]. \]

The resulting field is given by

\[ H(r) = -\text{grad} \; V(r). \]

From this expression we get, e.g., {cf. [1], eqs. (9) (13), (16), (21)}

\[ H_{\parallel}(q, \gamma) = -\frac{1}{\mu} \sum_{N} \frac{i\gamma v_{N}(q)}{\gamma^2 + \lambda_{N}^2} \int_{V_0} dV' \exp(-i\gamma z') \times \]

\[ \times \left[ M_{\parallel}(r') \cdot \text{Grad}' v_{N}(q') - i\gamma M_{z}(r') v_{N}(q') \right]. \]