Dynamics of Wire Boom Oscillations on a Spinning Satellite Part I, Lagrangian Equations of Motion and Transient Response

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1. Introduction

For measuring electric fields [1] and some plasma parameters [2] in the magnetosphere it is desirable to extend sensors some distance from the vehicle in order to reduce spacecraft-induced perturbations. Telescoping, semirigid booms are frequently used for this purpose. However, their length is often limited by weight, mechanical rigidity and structural stability considerations. A sensor on the end of a flexible cable may be deployed in the spin plane of the spacecraft where centripetal force tends to maintain it perpendicular to the spin axis. On the US Air Force Space Test Program Scientific Satellites S3-2 and S3-3, four sensors mounted on wire booms were successfully used for the first time to measure environmental electric field vectors. In addition to reducing spurious effects, increasing the length of the wire increases the magnitude of the potential difference between the ends of the wire. The sensor separations of up to 36.6 m permit a significant increase in the quality of the electric field data obtainable in the spin plane of the vehicle. Our purpose in this paper is to present the dynamic analysis of the oscillation of this four-wire boom system during deployment and in subsequent operation, in configurations of different symmetry.

Much of the previous work has been concerned with long, semirigid booms as reported for example by Cherchas [3], Cloutier [4], Josloff [5], and Underwood [6], and for which effects of solar induced dynamic perturbations have been noted to be important [7, 8]. Some aspects of wire boom dynamics have been reported for the case of a single cable counterweight for which the gravity induced normal modes along the length of the cable have been studied by Chobotov [9, 10] and Christ and Eisley [11]. Approximate wire boom computer simulations were done as part of the preliminary studies [12, 13] for the Small Scientific Satellite Series. The transverse higher mode oscillations along the length of a lumped mass model for the wire were shown by Weitzmann [14] to be insignificant, because of the high damping inherent in bending of the cable. Weitzmann showed that the significant wire response consisted of hinged
motion at the point of attachment and recommended installation of a damper (Coulomb). In this work we obtain expressions in closed form, (1) for a useful model that describes system motion during and after deployment, and (2) for the normal mode frequencies of oscillation of the boom system. This is useful both for minimizing potentially destructive excursions during deployment and for avoiding resonance with other satellite systems and modes of operation.

The model used for this analysis consists of a spinning rigid centerbody with four wires extending along perpendicular axes, normal to the spin axis. Each wire, of finite mass density $\rho$, has a small mass $m$ at its end and is hinged at its exit point, a distance $r_0$ from the spin axis. Subsequent to orbit insertion the wire booms are extended by a mechanism with control over the extension and retraction of opposite pairs of wires. The length deployed may be controlled by (a) a contact through the wire insulation at regular intervals, (b) a deployment timer which finally stops deployment should the contact be overridden, and (c) ground control in real time.

Our analysis starts with a Lagrangian of the complete system for motion within the spin plane. Our formulation takes into account wire boom damping, different boom lengths, different deployment rates, and centerbody translations that may be present with certain normal modes. From the Lagrangian seven nonlinear coupled equations of motion are derived, without small oscillation amplitude approximation. Computer generated solutions of these coupled equations simulate the time-dependent dynamical behavior of the complete system.

To gain physical insight into the transient response properties of the system due to boom extension/retraction, analytical solutions are obtained by means of Laplace transform techniques for a linearized model with equal boom lengths. The analytical results are compared with those of computer simulations. A systematic study of the normal modes of the system will be presented in a subsequent paper [15].

2. Lagrangian Dynamics in the Spin Plane

We consider the motion of the wire booms in the spin plane. Since the presence of an effective wobble damper [12] installed on the satellite virtually eliminates out-of-spin-plane boom oscillations, the in-plane oscillations are expected to be the most significant. We derive the Lagrangian in a coordinate system rotating at the spin frequency of the satellite hub. In this corotating frame in which the hub is at rest, the tip masses are positioned at $(x_i, y_i)$ where

\begin{align*}
x_i &= r_i \cos (\phi_i + \delta_i) + r_0 \cos \delta_i, \quad (i = 1, \ldots, 4); \\
y_i &= r_i \sin (\phi_i + \delta_i) + r_0 \sin \delta_i, \quad (i = 1, \ldots, 4)
\end{align*}

where $\delta_i = (i-1)\pi/2$, $r_i$ is the boom length as measured from the exit point, $r_0$ is the hub radius, and $\phi$ is the angular deflection of the boom as measured from equilibrium position as shown in Figure 1.