Virtuality evolution and rapidity structure of a simplified parton shower*

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Abstract. Continuing our research program on the origin of the Negative Binomial (NB) regularity in high energy collisions we discuss virtuality evolution and rapidity structure of a simplified parton shower (SPS) assuming essentials of QCD in a correct kinematical framework. We find that multiplicity distributions (MD) of final partons in full phase space and in symmetric rapidity windows are well fitted by NBMD's. Using generalized Local Parton Hadron Duality as hadronization prescription, NB regularity shows up also for final hadron MD's. Clan structure analysis manifests both at partonic and hadronic level very interesting properties which are consistent with previous findings in experimental data on MD's and in standard Monte-Carlo-Simulations. It is striking that in the SPS model the average number of clans scales with virtuality in a fixed rapidity window.

1 Introduction

This research program has been initiated in collaboration with Léon Van Hove in the framework of previous studies [1] on the origin of Negative Binomial (NB) regularity in final hadron multiplicity distributions (MD) in full phase space and in symmetric rapidity windows in high energy reactions [2] and in Monte-Carlo-Simulations both at hadron and parton level [3]. NB universality has been interpreted as an indication that a unified description of multihadron production in terms of QCD parton showers is possible [4]. Successful attempts to disentangle single QCD parton shower contributions have been also made using Monte-Carlo-Methods in $e^+e^-$ annihilation [5]. The very basic idea of our work is indeed that complex structures which we observe (like NB regularity) might very well be, at the origin of their evolution, elementary, and have simple properties. The higher degree of simplicity which we found in general at partonic level than at hadronic level agrees with this view. Guided by this criterion, in the present paper we focus our attention on a single shower initiated by a parton of given virtuality, which we propose to follow, in a correct kinematical framework, along its branchings, to the final parton level. The rapidity structure is assumed to be inspired by the essentials of QCD which we believe to be in the gluon self-interaction.

In Sect. 2 we describe the virtuality evolution of a simplified parton shower (SPS) model and we find integro-differential equations for the multiplicity distribution of the parton population. Their numerical solution in Sect. 3 leads to NB regularity in full phase space, at parton level, for ancestor parton virtualities between 50 and 10000 GeV. The rapidity structure of SPS is introduced in Sect. 4 and allows to study in Sect. 5 MD's in symmetric rapidity intervals at partonic level. It is remarkable that NB regularity shows up also in symmetric rapidity windows. Using then generalized LPHD prescription we describe the evolution of the parton shower into the hadron sector in Sect. 6. Some conclusions are drawn at the end.

2 Virtuality evolution of SPS

We study the elementary process

\[ W \rightarrow Q \rightarrow Q_0 + Q_1 \]

An initial parton of virtuality $W$ is degrading to virtuality $Q$ where it splits into two partons of virtuality $Q_0$ and $Q_1$ respectively. We require that $Q \geq Q_0 + Q_1$ and that $Q = 1$ GeV is the minimum allowed virtuality for a parton; this implies an automatic cut-off i.e. no splitting occurs for $Q < 2$ GeV.

We define the probability for a parton of virtuality $W$ to split at $Q$, $p(Q | W)$; this quantity is normalized to

\[ \int p(Q | W) dQ = 1 \]

The probability for a parton (ancestor) of virtuality $W$ to generate $n$ final partons is
The probability for a parton \( n = 1 \) which splits at virtuality \( Q \) to generate \( n \) final partons is 
\[ R_n(Q), \]
with \( R_n(Q) = \delta_{n1}. \)

A simple relation exists between the just defined probabilities
\[
P_n(W) = \int_{1}^{W} p(Q | W) R_n(Q) dQ.
\]

We assume that \( p(Q | W) \) can be factorized differently from what is usually done in the literature where scaling for \( p(Q | W) \) is introduced from the beginning [6]. By differentiating (1) with respect to \( W \) we obtain
\[
\frac{\partial P_n(W)}{\partial W} = P_n(W) R_n(W) + \int_{1}^{W} \frac{\partial p(Q | W)}{\partial W} R_n(Q) dQ.
\]

By taking into account (2) in the integral and by using the normalization condition on \( p(Q | W) \) one gets
\[
\frac{\partial P_n(W)}{\partial W} = P_n(W) \left[ R_n(W) - \int_{1}^{W} p(Q | W) R_n(Q) dQ \right].
\]

According to (1)
\[
\frac{\partial P_n(W)}{\partial W} = P_n(W) \left[ R_n(W) - P_n(W) \right].
\]

Now we express \( R_n(Q) \) in terms of the joint probability density of degrading from virtuality \( Q \) to virtuality \( Q_0 \) and \( Q_1 \), \( \mathcal{P}(Q_0 Q_1 | Q) \).
\[ R_n(Q) = \sum_{n_1=1}^{n-1} \int dQ_1 \mathcal{P}(Q_0 Q_1 | Q) R_{n-n_1}(Q_0) \]
\[ R_n(Q) \theta(Q - Q_0 - Q_1) \]
where the joint probability \( \mathcal{P}(Q_0 Q_1 | Q) \) is defined as follows
\[ \mathcal{P}(Q_0 Q_1 | Q) = p(Q_0 | Q) p(Q_1 | Q) N(Q) \]
\[ = C(Q)^2 p_0(Q_0) p_0(Q_1) N(Q) \]
with
\[ N(Q) = \left\{ \int_{1}^{\infty} C(Q)^2 p_0(Q_0) p_0(Q_1) \theta(Q - Q_0 - Q_1) dQ_0 dQ_1 \right\}^{-1}. \]

Define next the generating functions associated to \( P_n(W) \) and \( R_n(W) \)
\[ f(z, W) = \sum_{n=1}^{\infty} z^{n-1} P_n(W), \]
\[ g(z, Q) = \sum_{n=2}^{\infty} z^{n-2} R_n(Q). \]

for \( Q > 2 \text{ GeV}, R_n(Q) = 0 \) for \( Q < 2 \text{ GeV}, n \geq 2 \). The connection between the above two equations can be explored using Eqs. (5) and (6). We start by writing
\[ g(z, Q) = \sum_{n=2}^{\infty} z^{n-2} \mathcal{P}(Q_0 Q_1 | Q) R_{n-2}(Q_0) \]
\[ \cdot R_n(Q) \theta(Q - Q_0 - Q_1) dQ_0 dQ_1. \]

We divide the domain of integration of (11) into three subdomains. In the first subdomain \( [Q_0 < 2 \text{ GeV} \land Q_1 < 2 \text{ GeV}] \) no one of the two generated partons splits, in the second \( [1 \text{ GeV} < Q_0 < 2 \text{ GeV} \land (1 \text{ GeV} < Q_1 < 2 \text{ GeV} \land Q_0 > 2 \text{ GeV}) \land (1 \text{ GeV} < Q_1 < 2 \text{ GeV} \land Q_0 > 2 \text{ GeV}) \) only one of them splits, whereas in the third subdomain \( [Q_0 > 2 \text{ GeV} \land Q_1 > 2 \text{ GeV}] \) both partons are splitting. It follows that
\[ g(z, Q) = \sum_{n=2}^{\infty} z^{n-2} \mathcal{P}(Q_0 Q_1 | Q) \theta(Q - Q_0 - Q_1) dQ_0 dQ_1 \]
\[ + 2 \int_{1}^{\infty} dQ_0 \int_{1}^{\infty} dQ_1 \mathcal{P}(Q_0 Q_1 | Q) g(z, Q_1) \]
\[ \cdot \theta(Q - Q_0 - Q_1) \]
\[ + \int_{2}^{\infty} dQ_0 \int_{1}^{\infty} dQ_1 \mathcal{P}(Q_0 Q_1 | Q) g(z, Q_0) g(z, Q_1) \]
\[ \cdot \theta(Q - Q_0 - Q_1) dQ_0 dQ_1. \]

It results an integral non linear Volterra equation. The non linearity is of Lyapunov-Lichtenstein type [7]. By inserting now (1) into (9)
\[ f(z, W) = \sum_{n=1}^{\infty} z^{n-1} \int_{1}^{W} p(Q | W) R_n(W) dQ, \]

it follows that
\[ f(z, W) = \sum_{n=1}^{\infty} z^{n-1} \int_{1}^{W} p(Q | W) R_n(W) dQ + z \int_{1}^{W} g(z, Q) p(Q | W) dQ \]
or in differential form
\[ \frac{\partial f(z, W)}{\partial W} = p(W | W) [zg(z, W) - f(z, W)]. \]

This result allows us to calculate \( f(z, W) \) once \( g(z, Q) \) is known. The problem now consists in integrating Eqs. (12, 14, 15).

This problem can be solved in two relevant extreme cases, i.e. by limiting oneself to the domain where at most one of the two generated partons can split [domain A: \( \theta(Q - Q_0 - Q_1) \) is approximated in (12) by \( \theta(Q - Q_0) \times \theta(2 - Q_1) + \theta(Q - Q_1) \theta(2 - Q_0) \) or by extending the domain of integration in allowing \( Q_1 + Q_0 \) to be larger than \( Q \) [domain B: \( \theta(Q - Q_0 - Q_1) \) is approximated in (12) by \( \theta(Q - Q_0) \theta(Q - Q_1) \)]. Domain A and B represent of course unphysical situations. They are nonetheless very instructive for a better understanding of the structure of Eqs. (12) and (14) as will be seen later.

**Domain A.** Following (12) one gets
\[ g(z, Q) = 2N(Q) \int_{1}^{\infty} p(Q_0 | Q) dQ_0 \]
\[ \cdot \left\{ \sum_{n=2}^{\infty} z^{n-2} R_n(Q_0) \right\} \]
\[ + \left\{ \int_{1}^{\infty} p(Q_1 | Q) dQ_1 + z \int_{1}^{\infty} g(z, Q_1) p(Q_1 | Q) dQ_1 \right\}. \]