Light-cone structure of inclusive semileptonic $B$-meson decays

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Abstract. Investigating light-cone structure of inclusive semileptonic $B$-meson decays shows the meaning of the quark masses and the relevance of the transverse momentum distribution of the initial state quarks. The connection to the so called intuitive parton model and the equal velocity approximation is shown.

1 Introduction

Since the early days of electron-nucleon scattering experiments at SLAC [1, 2] and early theoretical papers on the parton model [3–5] more than twenty years have passed. In a previous work [6] the parton-model picture has been applied to inclusive semileptonic $B$-meson decays. In the present paper the light-cone structure of these decays is investigated with the help of simple field-theoretic methods. Therefore the hadronic tensor of $B$-decay is considered in configuration space. In the light-cone approximation the integration over the coordinates of the current commutator is known for a long time. One uses the technique of operator-product expansion [7] near the light cone. In deep inelastic scattering the commutator is approximated by its light-cone behaviour [8, 9] in Bjorkens scaling region [4]. This gives a relation between the Bjorken limit and light-cone dominance. For $B$-decays light-cone dominance is not necessarily a good approximation since the invariant square $q^2$ of the 4-momentum transfer and the scalar $P_B q$ have an upper limit given by the mass squared of the $B$-meson $M_B^2$. Terms proportional to heavy quark masses probably aren't any more negligible.

In order to investigate this in more detail in Sect. 2 the current commutator has been integrated out in its whole configuration space not only on the light cone. A model is presented in Sect. 3 in order to calculate the structure functions originating from the hadronic tensor. This model is called fieldtheoretical parton model (FPM) and compared with the intuitive parton model (IPM) of an earlier paper [6]. Within the frame of these investigations the meaning of the quark masses taking part in the decay and the relevance of a transverse momentum distribution of the $b$-quark will become clear which was an open question up to now. Furthermore a new contribution to the intuitive parton model shows up -- the so called negative $k_{0}$-region -- which is discussed in Sect. 4. The conclusions are summarized in Sect. 5. Details of the long calculations are given in [10].

2 The hadronic tensor

The inclusive semileptonic decay of $B$-mesons (Fig. 1)

$$B \rightarrow X_f e^{-} \bar{\nu}$$

(2.1)

is given by the decay of its heavy $b$-quark. The index $f = u, c$ labels the flavour of the final hadronic state. The decay is mediated by the flavour changing charged quark current $j_b$:

$$j_b(x) = V_{fb}^{*} \bar{q}(x) \gamma_{\mu} P_L \gamma_{\mu} q(x);$$

(2.2)

$$P_L = \frac{1 - \gamma_5}{2}.$$  

(2.3)

In analogy to deep inelastic neutrino-nucleon scattering the inclusive semileptonic width of the $B$-meson $\Gamma_f$ for $b \rightarrow f$ transitions is splitted into its leptonic $\Gamma_L$ and hadronic $\Gamma_H$ tensor. In lowest order:

$$\Gamma_f = \frac{G_F^2}{8\pi^3E_B} \frac{1}{2E_x} \frac{1}{2E_x} W_{\mu\nu}\Gamma_L^{\mu\nu},$$

(2.4)

$$\Gamma_L^{\mu\nu} = 2[p_\mu p_{\nu} + p_{\mu} p_{\nu} - g^{\mu\nu} p_x p_y + i e^{\mu\nu\rho\sigma} p_{\rho} p_{\sigma}].$$

(2.5)

$$W_{\mu\nu} = \sum_{n_f} \int \prod_{i=1}^{n_f} \left[ \frac{d^3p_i}{(2\pi)^32E_i} \right] (2\pi)^3 \delta^{(4)}\left(P_B - q - \sum_{i=1}^{n_f} p_i\right) \times |B|_{f,0} < n_f |j_{f,0}|n_f >.$$  

(2.6)

$\sum_{n_f}$ sums up all final hadronic states which consist of $n_f$ particles. The propagator of the intermediate $W$-boson has been neglected. Using the completeness relation of the final hadronic state, the 4-momentum conservation and
Fig. 1. Inclusive semileptonic decays of the B-meson from the parton model point of view. The index f = u, c labels the flavour content of the hadronic final state.

The translational properties of operators, one gets the well known result for the hadronic tensor:

\[ W_{\mu
u,f} = \frac{1}{2\pi} d^4x e^{iqx} \langle B| [j_{\mu,f}(x), j^*_{\nu,f}(0)] B \rangle. \]  

(2.7)

Its form factor decomposition is given below:

\[ W_{\mu
u,f} = -g_{\mu\nu} W_{1,f} + \frac{P_{B0} P^\nu_0}{M_B^2} W_{2,f} - i g_{\mu\nu} q^\nu \frac{P_{B0} P^\rho}{M_B^2} W_{3,f} \]
\[ + \frac{q_\mu q_\nu}{M_B^2} W_{4,f} + \frac{P_{B0} q_\nu + q_\rho P_{B0}}{M_B^2} W_{5,f} \]
\[ + i \frac{P_{B0} q_\nu - q_\rho P_{B0}}{M_B^2} W_{6,f}. \]  

(2.8)

The form factors \( W_{i,f} (i = 1, \ldots, 6) \) depend on the relativistic invariants \( q^2 \) and \( P_{B0} q \). If the lepton masses are neglected the contraction of the leptonic tensor with \( q^\mu \) or \( q^\nu \) disappears. In this case only \( W_1, W_2 \) and \( W_3 \) contribute to the width:

\[ \Gamma_f = \frac{G_F^2}{4\pi^2 E_F} \left[ \frac{d^3 p_e}{2E_e} \frac{d^3 p_B}{2E_B} \right] \left\{ q^2 W_{1,f} + \left( \frac{2(p_e P_B)(p_B P_B)}{M_B^2} - q^2 \right) \right\}. \]  

(2.9)

In case of massless leptons the phase space is given by the range of the invariant mass of the hadronic final state \( M_x \) corresponding to \( b \rightarrow u(b \rightarrow c) \) transitions:

\[ M_x(M_B) \leq M_x = \sqrt{(P_B^2 - q^2)} \leq M_B. \]  

(2.10)

In the following the hadronic tensor will be determined in case of quasifree quark fields. Quasifree quark fields obey the free Dirac equation with an effective mass \( m \). Evaluating the matrixelement of (2.7) with the help of (2.2) one gets:

\[ \langle B| [j_{\mu,f}(x), j^*_{\nu,f}(0)] B \rangle = -|V_{ub}|^2 (s_{\mu\nu\lambda} - i\epsilon_{\mu\nu\lambda}) \]
\[ \langle B| \bar{\Psi}_B(0) \gamma_\mu P_L \Psi_B(x) \mid B \rangle \partial^\alpha D(x, m_f), \]  

(2.11)

with

\[ s_{\mu\nu\lambda} = g_{\mu\nu} g_{\lambda\rho} - g_{\mu\rho} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\rho}, \]
\[ D(x, m_f) = \frac{\epsilon(b^0)}{2\pi} \left[ \delta(x^2) - \Theta(x^2) m_f^2 J_1(m_f \sqrt{x^2}) \right]. \]  

(2.12)

(2.13)

where \( J_n(x) \) is the Bessel function of order \( n \). The Pauli-Jordan function \( D(x, m_f) \) is defined by the anticommutator of the free fermionic field \( \Psi(x) \) (see for example [11] or [12]):

\[ \{ \Psi(x), \bar{\Psi}(y) \} = -i(\gamma^{\mu} \partial_{\mu} - \gamma^5) \delta(x - y, m). \]  

(2.14)

In (2.13) the expression proportional \( \delta(x^2) \) recovers the common light-cone contribution. The part proportional the final state quark mass \( m_f \) describes contributions from outside the light cone \( (x^2 > 0) \). With increasing \( m_f \) these contributions are expected to become more and more important. The hadronic matrix element on the right hand side of (2.11) is decomposed in Lorentz-covariants:

\[ Q^2 = \langle B| \bar{\Psi}_B(0) \gamma^5 P_L \Psi_B(x) \mid B \rangle = P^\mu_B F^\mu_1(x^2, P_B x) \]
\[ + x^2 F_2(x^2, P_B x). \]  

(2.15)

Afterwards the Fourier transforms of \( F_j (j = 1, 2) \) are introduced

\[ F_j(x^2, P_B x) = \int_0^\infty d\xi_1 \int_{-\infty}^\infty d\xi_2 e^{i\xi_1 p_x - i\xi_2 \xi_2} M^\xi_2 \xi_2^2 G_j(\xi_1, \xi_2), \]  

(2.16)

and (2.11), (2.15) and (2.16) are inserted into the expression of the hadronic tensor (2.7). The integration over the spatial coordinates has been done analytically. Details of the computation are given in [10]. The result is given below:

\[ W_{\mu\nu,f} = |V_{ub}|^2 (s_{\mu\nu\lambda} - i\epsilon_{\mu\nu\lambda}) \int_{-\infty}^\infty d\xi_1 \]
\[ \times \int_{-\infty}^\infty d\xi_2 \left[ P^\mu_B G_1 + G_2 \frac{\partial}{\partial k_2} \right] \left[ A^\nu_f + B^\nu_f \right]. \]  

(2.17)

with

\[ A^\nu_f = k^\nu \exp \left( i \frac{\xi_2^2}{4\xi_2^2} \left[ 1 - i \frac{\xi_2^2}{4\xi_2^2} + i \frac{2\xi_2 M_B^2}{\xi_1 M_B - P_B q} \frac{\partial}{\partial k_2} \right] \right) \times \varepsilon(k_0) \Theta(k^2), \]
\[ B^\nu_f = i \frac{\xi_2^2}{4\xi_2^2} k^\nu \exp \left( i \frac{k^2 + m_f^2}{4\xi_2^2 M_B^2} \right) J_2 \left( \sqrt{\frac{\xi_2 k^2}{4\xi_2^2 M_B^2}} \right) \times \varepsilon(k_0) \Theta(k^2), \]
\[ \xi_2^2 = \frac{m_f^2}{M_B^2}, \]
\[ k^\nu = \xi_1 P_B^\nu - q^\nu. \]  

(2.18)

(2.19)

(2.20)

(2.21)

The hadronic tensor consists of two parts. The expression proportional to \( A^\nu_f \) contains the light-cone contribution plus some final state mass effects. \( B^\nu_f \) corresponds to the off light-cone region, sensitive to the final state quark mass. It will not contribute in the case of a massless final state quark. All information about the hadronic structure of the \( B \)-meson is contained in the structure functions \( G_1 \) and \( G_2 \). In a specific model they can be calculated.

Considering the massless limit of the final state quark \( (m_f \rightarrow 0) \), only the expression containing \( A^\nu_f \) contributes with \( \xi_2 = 0 \). The \( \xi_1 \)-integration is performed with the \( \delta(k^2) \) function. The condition \( k^2 = 0 \) results in two values for \( \xi_1 : \)

\[ \xi_1 = \frac{P_B q}{M_B^2} \pm \sqrt{\frac{(P_B q)^2 - q^2}{M_B^2 - M_B^2}}, \]  

(2.22)