TWO QUARK EXCITATIONS IN TWO NUCLEON SYSTEM*)

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The unambiguously determined bases of the finite dimensional irreducible representation of the unitary and orthogonal groups are constructed. It is shown that the bases can be successfully applied in finding decomposition coefficients when the system of physical particles is divided into two subsystems. The general forms for some of these coefficients are presented.

Recently a part of theoretical nuclear physics studying the lightest nuclei have been influenced by the hadron quark model ideas. In the nonrelativistic approximation properties of a few nucleon system and the NN-interaction at small distances (~0.5—0.7 fm) are investigated considering nucleons as systems of quarks [1, 2]. In this approximation the properties are determined by the quark structure of the nucleons which can be considered, owing to the peculiarity of the qq-interaction, as quark clusters. The existence of these clusters allows us to use the algebraic apparatus of the nucleon cluster model [3]. The comparatively large number of quarks and additional group of symmetry-SU3 colour make the generalization of known methods nontrivial. For example, only S-state of completely symmetric three-quark system is taken into account for the two nucleon problem [4—8], and the role of the other states is investigated in rough approximation [9]. This leads to contradictions in the results of different authors.

In ref. [10] the general method of construction of unitary group irreducible representation (IR) basis functions is proposed. The method is based on using of the Young scheme (YS) formalism, which was successfully applied to construction of the IR of groups, composition rules [11], Casimir operators [12], etc. It is possible to build a basis of tensor IR of the orthogonal groups as well as of the unitary groups. Let us designate the YS for IR of the unitary groups by {2} and for the IR of the orthogonal groups by [2]. (2) — (21 22 ... 2m) is the partition of the number m, which shows the tensor rank of the IR. The basis functions of the unitary group IR (2} have the form [10]

\[ |\{\lambda\}, \phi\rangle = N \cdot J^{(\lambda)} f \phi \]

N is the normalization coefficient. The factor \( \phi \) completely determined by the group \( U_n \) is a product of one-particle states which form the basis functions of the fundamental IR \{1\}. Let us call it \( \phi \) projection. The order of these states in the product is determined by the rule of filling in the YS \{\lambda\} for obtaining the standard Young tableaus. The operator \( J^{(\lambda)} f \) in (1) is the projection operator of the standard Young-Yamanouchi IR of the permutation group \( S_m \), and is universal for \( U_n \) and \( O_n \) groups:

\[ J^{(\lambda)} f \equiv J^{(1)} f \).

Here the index \( f \) stands for Yamanouchi symbol of the \( S_m \) IR \{\lambda\}. This operator permutes the one-particle states of \( \phi \). The convenient operator form for this projector was found in [10].

The generalization of the method for the orthogonal group is not simple. In contrast with the group \( U_n \) it is possible that for the group \( O_n \) the number of nodes in the YS \{\lambda\} to be less than a rank of the tensor IR. Therefore the basis function of IR depends on the \( U_n \) group YS \{µ\}, the contraction of which gives us the YS \{\lambda\}. Two or more \{\lambda\} can appear after contraction. One needs to distinguish them. It is well known [13] that mathematical chain of groups \( U_n \supseteq U_{n-1} \supseteq \ldots \supseteq U_1 \) is presented at the symposium "Mesons and Light Nuclei", Bechyně, Czechoslovakia, May 27—June 1, 1985.

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$\ldots \supset U_1$ is completely determined by Gelfand-Ceytlin's basis $[x_n, x_{n-1}, \ldots, x_1]$, where $x_i = (m_{1i}, m_{2i}, \ldots, m_{ni})$ is a signature of the $U_i$ subgroup IR. Arbitrary integers $m_{ij}$ satisfy the condition $m_{1j} \geq m_{2j} \geq \ldots \geq m_{nj}$. Therefore one can use the number of nodes in rows of the YS $[\lambda] = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, as the parameters $m_{ij}$. The YS $[\lambda], [\lambda'], [\lambda''], \ldots$ have consequently $m_1 (m-1), m_2 (m-2), \ldots$ nodes and Yamanouchi symbol $j$ shows us which nodes must be unhooked. Therefore the basis (1) is the Gelfand-Ceytlin's basis. One can easily find the number of parameters unambiguously characterizing the functions of the mathematical chain. Actually,

$$N_u = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \tag{3}$$

In the case of the physical chain

$$U_n \supset O_n \supset \ldots \supset O_1 \tag{4}$$

the number of parameters decreases for the number of rows in the YS $[\lambda]$ and cannot be larger than

$$v = \langle n/2 \rangle = \frac{n - \text{mod } n}{2}. \tag{5}$$

The parameter $v$ is determined by (5).

The construction of the projection $\varphi$ is simple for the group $SU_3$. One must start from the senior vector which is obtained from YS $[2]$ by filling the first row with indices $v$, the second row with indices $(v-1)$, etc. Subtracting 1 from each index of the previous vector and summing up all obtained tableaus gives us the next vectors. The projector acts on this sum.

If one needs basis functions with fixed permutation symmetry $\{F\}_k$, the operator $J_k^{(F)}$ will project this state from the function (8):

$$\{F\}_{ki} [\{\mu\}_j (\{\lambda\} \varphi, \{x\} \psi)] = N'J_k^{(F)} [J_1^{(\lambda)}(J_1^{(\lambda)} \varphi, J_1^{(\lambda)} \psi)]. \tag{9}$$

It is easy to use the function (9) for calculation of the two quark excitation in the six-quark system. One can calculate matrix elements of physical operators, weights of quark system states, etc. All these problems require that the quark system should be divided into subsystems. In terms of the group theory the division is connected with reductions of the IR of a group $G$:

$$\{\lambda\}_G^G \supset \{\lambda_1\}_G^G \times \{\lambda_2\}_G^G, \tag{10}$$

where $\text{mod } n = 1$ for odd $n$ and $= 0$ for even $n$. Then the number of parameters for the chain (3) is

$$N_{u=0} = n + \sum_{i=1}^{n} \langle n/2 \rangle = \frac{n^2 + 4n - \text{mod } n}{4}. \tag{6}$$

It is possible to make up the lack of parameter numbers

$$\Delta = N_u - N_{u=0} = \frac{n^2 - 2n + \text{mod } n}{2}. \tag{7}$$

by introducing contracting tensor type $\{x\}$ and so the basis function for the group $O_n$ has a form

$$\{\mu\}_j : (\{\lambda\} \varphi, \{x\} \psi) = N'J_k^{(\lambda)}(J_1^{(\lambda)} \varphi, J_1^{(\lambda)} \psi). \tag{8}$$

The projector $J_k^{(x)}$ acts in the space of vectors with contracted indices, $J_1^{(\lambda)}$ acts in the space of the rest vectors and $J_j^{(x)}$ mixes these two spaces and builds tensor of the type $\{\mu\}$. The projection $\psi$ is a product of polynomials $\sum_{t} (-1)^{t+1} x_2^t x_3^{-t}$ where $x_t$ stays for the basis function of fundamental IR of the orthogonal group and $t$ takes on the values $v, (v-1), \ldots -(v-1), -v. \tag{11, 12}$

$$\{\lambda\}^{Gn+m} \supset \{\lambda_1\}^{Gn} \times \{\lambda_2\}^{Gm}, \quad \{\lambda\}^{Gnm} \supset \{\lambda_1\}^{Gn} \times \{\lambda_2\}^{Gm}. \quad \text{Czech. J. Phys. B 36 [1986] 355}$$