ON THE ASYMPTOTIC-TIME SYMMETRY BREAKING 
IN THE SYMMETRIC SPIN-BOSON MODEL*)

V. Čápek, P. Chvosta

Institute of Physics of Charles University, Faculty of Mathematics and Physics,
Ke Karlovu 5, 121 16 Prague 2, Czechoslovakia

Received 21 April 1989

The symmetric spin-boson model is treated using the time-convolution Generalized Master Equations with two forms of the Peier projection superoperator $D$ and for the Ohmic type of coupling. With the form of $D$ usually used in the strong coupling regime and using the memory kernel exact to the second order but formally summed up partially to the infinity, standard results for the asymptotic-time symmetry breaking at zero temperature are obtained for relaxed initial condition. With the second form of $D$ usually used in the weak coupling regime (but applicable in general) and with formally the same accuracy of the memory kernel but for unrelaxed initial condition, no asymptotic time symmetry breaking is obtained.

1. Introduction

Since 1982, the spin-boson model with the Hamiltonian

$$H = H_s + H_R + H_{s-R}$$

$$=-\frac{1}{2} \hbar \Delta \sigma_x + \sum_{i=1}^{M} \hbar \omega_i b_i^+ b_i + \frac{1}{2} \hbar \sum_{i=1}^{M} k_i \sigma_z (b_i^+ + b_i) ,$$  \hspace{1cm} (1a)

$$\sigma_x = a_1^+ a_2 + a_2^+ a_1 = (|1\rangle \langle 2| + |2\rangle \langle 1|) \otimes 1_R ,$$  \hspace{1cm} (1b)

$$\sigma_z = a_1^+ a_1 - a_2^+ a_2 = (|1\rangle \langle 1| - |2\rangle \langle 2|) \otimes 1_R$$  \hspace{1cm} (1c)

attracts attention of theoreticians [1-3]. Already Bray and Moore [2] turned attention to the fact that after taking the thermodynamic limit ($M \to \infty$), this symmetric model ($1 \leftrightarrow 2$, $b_1 \leftrightarrow -b_1$ etc.) might yield an asymmetric particle distribution in the asymptotic time domain owing to initially asymmetric particle distribution. Kinetic studies based on time-convolution or time-convolutionless Generalized Master Equations (GME) with initially relaxed as well as unrelaxed boson cloud around the particle located at $t = 0$ in e.g. the “left” state $|1\rangle = a_1^+ |\text{vac}\rangle$ mostly

*) Dedicated to Professor J. Kvasnica on the occasion of his sixtieth birthday.
supported this opinion. Namely, it was concluded that in e.g. the case of the Ohmic coupling with the strength function

\[ \gamma(\omega) = \lim_{M \to \infty} \sum_{i=1}^{M} k_i^2 \delta(\omega - \omega_i) = \gamma \omega + o(\omega), \quad \omega \to 0, \]  

there is no such asymptotic-time symmetry breaking at non-zero temperatures \( T > 0 \) or at \( T = 0, \gamma \leq 2 \) while at \( T = 0, \gamma > 2 \), the symmetry breaking exists [4–8]. Unfortunately, these studies are mostly limited to the lowest order in \( \Delta \) – the fact which makes all such theories unreliable in the long-time domain. This also explains why these conclusions do not fully agree with the second-order theory of Kassner [9].

Another typical feature of the contemporary GME theories of the symmetry breaking in our model is that they use the Argyres and Kelley [10] projection superoperator preserving diagonal as well as off-diagonal elements of the density matrix of the particle. The aim of the present work is to show that with the diagonalizing Peier [11] projection superoperator and using some previously derived results, the theory of the asymptotic-time symmetry breaking based on the time-convolution GME becomes surprisingly simple. With the relaxed initial condition and \( T = 0 \), we reproduce the standard results. With unrelaxed initial condition and \( T = 0 \), however, this theory yields no asymptotic-time symmetry breaking. In both cases, previously derived approximate forms of the memory kernel (to be discussed below) are used.

2. Relaxed initial condition


\[ (D \ldots)_{a_i b_i} = \delta_{ab} \delta_{a_i b_i} \sum_{\lambda} s_{a_i b_i} \lambda \cdot \]  

Here \( p_\lambda \) is the Boltzmann weight of the \( \lambda \)-th eigenstate of \( H_R + \hbar \sum_{i=1}^{M} k_i (b_i^+ + b_i^-) \) and the Latin (Greek) indices correspond to eigenstates

\[ |1\lambda\rangle = \prod_{i=1}^{M} \left[ \frac{1}{(\lambda_i!)^{1/2}} \left( b_i^+ + \frac{1}{2} k_i (\omega_i)^{\lambda_i} \right) \exp \left[ \sum_j^{1/2} k_j (\omega_j) (b_j - b_j^+) \right] \right] \]

\[ \times a_i^+ |\text{vac}\rangle = |1\rangle \otimes \prod_{i=1}^{M} \left[ (\lambda_i!)^{-1/2} \left( b_i^+ + \frac{1}{2} k_i (\omega_i)^{\lambda_i} \right) \right] |\text{vac}\rangle \]

\[ = |1\rangle \otimes |\lambda\rangle \]  

(4a)

\[ |2\lambda\rangle = \prod_{i=1}^{M} \left[ \frac{1}{(\lambda_i!)^{1/2}} \left( b_i^+ - \frac{1}{2} k_i (\omega_i)^{\lambda_i} \right) \exp \left[ - \sum_j^{1/2} k_j (\omega_j) (b_j - b_j^+) \right] \right] \]

\[ \times a_i^+ |\text{vac}\rangle = |2\rangle \otimes \prod_{i=1}^{M} \left[ (\lambda_i!)^{-1/2} \left( b_i^+ - \frac{1}{2} k_i (\omega_i)^{\lambda_i} \right) \right] |\text{vac}\rangle \]  

(4b)