Recently reported electrochemically induced nuclear fusion of deuterium dissolved in palladium [1] renewed the old interest in tunnelling. The reason is simple - without a drastic increase of tunnelling efficiency (as compared to that given by the elementary quantum mechanics) through the Coulomb barrier separating two D-nuclei, there is no practical possibility for the fusion to take place at room temperature. In this work, irrespective of the final result of cold fusion experiments, we want to argue that a mechanism of the excited-state-induced bath-assisted tunnelling (which does not exist in vacuum but does in solids [2 - 4]) might in principle be able to increase low-energy values of the tunnelling transmissions coefficient by many orders of magnitude (almost six hundred orders, i.e. by $\approx 1400$ in the exponent in our example treated below). Surprisingly, this new channel may be traced in higher orders of the usual perturbation theory but has been currently overlooked as it falls out, e.g., in the Haken-Strobl-Reineker parametrization of the Stochastic Liouville equation model [5].

Let us assume the spherically symmetric single-particle model with the potential energy

$$V(r) = \begin{cases} -V_0 = \text{const}, & 0 \leq r \leq r_0 = 10^{-15} \text{m}, \\ e^2/(4\pi\varepsilon_0 r), & r_0 < r \leq a \end{cases}$$

where $a$ is of the order of the lattice constant. Elementary quantum mechanics yields for the tunnelling transmission coefficient

$$D(E) \approx \exp\left\{ -\frac{2}{\hbar} \int_{r_0}^{a} \frac{e^2/(4\pi\varepsilon_0 E)}{2m(V(r)-E)} \frac{1}{3} \, dr \right\} = \exp\left\{ -\frac{e^2}{2\pi\hbar\varepsilon_0\sqrt{V}} \left[ \arctg\sqrt{\frac{x-1}{x}} - \frac{\sqrt{x-1}}{x} \right] \right\}, \quad x = \frac{e^2}{4\pi\varepsilon_0 r_0 E}$$

where $E$ is the energy of the incident particle. Taking for $m$ the deuteron mass and $E = 1 \text{ eV}$ for the energy we obtain

$$D(E)|_{E=1\text{eV}} \approx \exp(-1403.6) \approx 2.66 \times 10^{-610}.$$  

The fact that this value is practically zero is due to the width and height of the potential barrier ($e^2/(4\pi\varepsilon_0 r_0) = 1.44 \text{ MeV}$).

In [2 - 4], we have shown that a quantum particle in a symmetric double well interacting with a bath performs coherent oscillations between the two wells (1 and 2) with the frequency

$$\Omega \approx \frac{1}{\hbar\varepsilon} \sum_{\lambda,\mu,\nu} \mathcal{H}_{1\nu,3\mu} \mathcal{H}_{3\mu,2\lambda} e^{R\alpha\beta}_{\lambda\nu}$$

as far as standard tunnelling and overbarrier channels become sufficiently suppressed. Here, like in [4], $\mathcal{H}$ is the Hamiltonian of the particle-bath interaction, the Greek indices designate the bath states, $e^{R\alpha\beta}_{\lambda\nu}$ is the initial bath density matrix and the index 3 denotes the first particle-excited state having sufficient overlap with both the left hand (i.e. for $r \leq r_0$) (1)
and the right hand (2) well states; \( \bar{\varepsilon} \) is then the corresponding particle excitation energy. The assumption of symmetry of the double-well potential is not needed here. We rather need approximate equality of the two single-well energies. Although even this requirement may be lifted in a way, we assume (for simplicity) \( V_0 \) to be fitted in such a way that the particle in the left hand well has a well-defined (excited i.e. intermediate) state with energy \( E = 1 \text{ eV} \) (i.e. of the same order as the energy of the D-particles in the Pd lattice). For \( 1 \leftrightarrow 2 \) transitions, virtual transitions to higher particle states are responsible. There is no energy conservation law limitation for such transitions as the transitions are not real [4]. Hence, though the phonon (or electron) bath energies are quite low, we can take \( \bar{\varepsilon} \) as high as \( 10^5 \text{ eV} \) because

\[
D(E)|_{E=10^5}\text{eV} \approx 0.05 .
\]

Therefore, the existence of such a state with appreciable overlaps with both the left and the right hand well state may be anticipated. It is worth mentioning, however, that more such excited states may additively contribute to (4); for simplicity, we ignore this possibility here. Taking the strength of the coupling to the bath as \( 0.01 - 1 \text{ eV} \) (typical coupling strength to phonons) and approximating the matrix elements of \( \mathcal{H} \) in (4) as this strength multiplied by a corresponding overlap, we have from (4)

\[
\Omega \approx 1.52 \times 10^6 s_1 s_2 \text{ to } 1.52 \times 10^{10} s_1 s_2
\]

where \( s_1 \) and \( s_2 \) are overlaps of the state 3 with the left (1) and right (2) well states. Taking roughly \( |3| \) as a plane wave state (here we use the fact that in Pd, D moves as quasifree), we can calculate the overlaps with the result (for \( a = 10^{10} \text{m} \))

\[
\begin{align*}
s_1 &\leq 2.8 \times 10^{-3} , \\
s_2 &\leq 10^{-4} .
\end{align*}
\]

Thus, we obtain

\[
\Omega \leq 4.3 \times 10^{-1} \text{ to } 4.3 \times 10^{+3} \text{s}^{-1} .
\]

The attempt frequency (of the particle in the right hand well state to get through the barrier) is

\[
\omega_{at} = 2 \pi \frac{v}{2a} = \frac{\pi}{a} \sqrt{\frac{(2E/m)}{E=1\text{eV}, a=10^{-10}\text{m}}} \approx 3.1 \times 10^{14} \text{s}^{-1}
\]

(here \( v \) is the mean deuteron velocity), i.e. the renormalized transmission coefficient \( D_{\text{ren}} \) for deuterons with energy \( E = 1 \text{ eV} \)

\[
D_{\text{ren}} = \frac{\Omega}{\omega_{at}} \leq 1.4 \times 10^{-15} \text{ to } 1.4 \times 10^{-11} .
\]

These surprisingly great values as compared to the lowest order (in the deuteron-bath coupling) result (3) are due to the fact that \( D_{\text{ren}} \) does not contain any exponentially small overlap of two low-energy states in the individual wells. It illustrates the important role which the bath can play in tunnelling; this conclusion is independent of the final result fusion experiments in solids may have.

Let us discuss in conclusion several mechanisms which may influence the result of our simplified calculation. First, we have assumed (1), which is a spherically symmetric model.