QUANTUM STATISTICS OF RADIATION IN THERMOSTAT*)

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The influence of thermal noise on subpoisson statistics of the one-mode field is researched. The equilibrium distribution of subpoisson states is shown to have some subpoisson properties. The subpoisson effect in the evolution of the superposition of coherent signal and thermal noise depends on the field frequency, temperature and initial signal amplitude.

The present paper is dedicated to the memory of Prof. Marian Gmitro, a well-known specialist in nuclear physics untimely passed away. Being a man of great learning, in recent years he displayed an increased interest in achievements of quantum optics, and in particular, in their possible application to nuclear physics problems.

I. Introduction

Quantum electrodynamics usually treats the Fock state or a coherent state of the electromagnetic type. Statistical physics also considers a chaotic state, an equilibrium state of photons at temperature $T$. In recent years, much attention has been paid to theoretical and experimental study of the so-called "nonclassical" states of electromagnetic fields [1 – 3].

Let us explain which states of the Bose field are called nonclassical. For this purpose we shall consider the one-dimensional quantum harmonic oscillator that is known to be the simplest model of the one-mode electromagnetic field with frequency $\omega$,

$$h = \omega a^+ a, \quad [a, a^+] = 1, \quad h|n\rangle = \omega n|n\rangle.$$ (1)

Here $|n\rangle = (n!)^{-1/2} (a^+)^n |0\rangle$ is the Fock state, $a|0\rangle = 0$. The coherent state $|\alpha\rangle$ satisfying the condition

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

is usually introduced by the relation [5]

$$|\alpha\rangle = D(\alpha) |0\rangle,$$

$$D(\alpha) = \exp (\alpha a^+ - \alpha^* a)$$

*) Dedicated to the memory of M. Gmitro.
and is connected with the Fock representation by the expression

\[
|\alpha\rangle = e^{-\langle 1/2 \rangle |\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} |n\rangle .
\]

A squeezed coherent state determined by the relation [1–3]

\[
|x, \xi\rangle = D(x) S(\xi) |0\rangle ,
\]

\[
S(\xi) = \exp \frac{1}{2} (\xi a^2 - \xi^* a^+ 2)
\]

(2)
can serve as an example of nonclassical states. An important property of a state of that type is the reduction of quantum noise. Indeed, let us consider the operators of the field quadratures

\[
x = (a^+ + a)/2 , \quad y = i(a^+ - a)/2
\]

which are in fact generalised dimensionless coordinate and field momentum. Obviously,

\[
[x, y] = \frac{i}{\hbar}.
\]

Therefore, the relation of Heisenberg uncertainties has the form

\[
\langle (\Delta x)^2 \rangle \langle (\Delta y)^2 \rangle \geq \frac{1}{16} .
\]

(3)

In averaging for a coherent state

\[
\langle (\Delta x)^2 \rangle = \langle (\Delta y)^2 \rangle = \frac{1}{4}
\]

and in relation (3) an exact equality is realised (the state of minimal uncertainty). For the squeezed coherent state

\[
\langle (\Delta x)^2 \rangle = \frac{1}{4} \left( \text{ch} \ 2r - \text{sh} \ 2r \cos \varphi \right),
\]

\[
\langle (\Delta y)^2 \rangle = \frac{1}{4} \left( \text{ch} \ 2r + \text{sh} \ 2r \cos \varphi \right),
\]

\[
\xi = r e^{i\varphi} .
\]

It is clear that the condition cos 2\varphi = 1 also realises the state of minimal uncertainty but with an asymmetric distribution of quantum fluctuations

\[
\langle (\Delta x)^2 \rangle \neq \langle (\Delta y)^2 \rangle .
\]

In other words, the compression of quantum noise takes place [1–3]. Another important class of nonclassical states are states with sub-Poisson statistics of quanta for which

\[
\langle (\Delta a^+ a)^2 \rangle < \langle a^+ a \rangle .
\]

(4)