SEMISTATE IMPLEMENTATION: DIFFERENTIATOR EXAMPLE*

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Abstract. It is shown that the semistate equations can be transformed via a linear transformation into a form which is useful for physical realizations. The result is applied to the example of a semistate described differentiator which is then realized through an op-amp circuit composed of integrators.

1. Introduction

Recently there has been a large interest in the semistate theory of circuits [1]-[7]. This is because the semistate description is a natural one for circuits which is also very general and contains the state variable one as a special case when the latter exists. The semistate equations for nonlinear time-varying circuits were described in [1] in the canonical form of

\[ \mathbf{A} \frac{d\mathbf{x}}{dt} + \mathbf{B}(x,t) = \mathbf{D} u \]  
\[ y = \mathcal{L} x \]

where \( u \) = input, \( y \) = output, \( x \) = semistate, and \( \mathbf{A}, \mathbf{D}, \mathcal{L} \) are constant matrices. \( \mathbf{B}(\ldots) \) is a nonlinear time-varying operator. In the linear case many authors have considered the solution of the semistate equations, where the equivalent standard canonical form is used in the analysis [3]-[5]. In a somewhat different approach the Drazin inverse has been used in the analysis of such systems to obtain the solutions in “one fell swoop” [5], [6]. However, except in [2], semistate theory has not been used in the design of circuits. This is in contrast to one of the distinct advantages resulting because of

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possibility of realizing state variable designs by integrating all of the state variables via integrator circuits. Here we show that the same type of advantage should be available within semistate theory, thus opening up the possibility for design via the canonical semistate equations of a system.

In particular, in this paper we will show that through a linear transformation it is also possible to obtain an equivalent representation of the semistate equations which is useful in the physical realization of semistate described systems. The resultant equivalent representation can be used in the construction of semistate described systems by incorporating integrators on all of the semistate variables. The details of this transformation are given in Section 2. The results are then used in the realization of a differentiator example which is discussed in Section 3 and 4 with simulation details given in Section 5.

2. Transformation

We consider the canonical semistate equations of Equation (1). First we premultiply by a nonsingular matrix $P$. Second, we change the semistate variable by a linear nonsingular transformation of matrix $Q$

$$x = Qx.$$  \hspace{1cm} (2)

Finally we add $dx/dt$ to both sides of the result. Letting $I$ denote the identity the transformed equations can be arranged into the form

$$\frac{dx}{dt} = (I - P\otimes Q)\frac{dx}{dt} - P\otimes (Qx, t) + P\mathcal{D}u$$  \hspace{1cm} (3a)

$$y = \mathcal{L} Qx$$  \hspace{1cm} (3b)

Since the transformations which brought (1) to (3) can all be inverted, it is clearly possible to transform (3) back to (1). Consequently, the relationship between the representations (1) and (3) is an equivalence relationship preserving the input and output map and under it we consider that (1) and (3) are equivalent. This equivalence contains a number of others in literature [7], [8] and the freedom to choose among all possible $P$ and $Q$ gives considerable latitude in the circuit structure allowing us to look for ones with certain desirable properties, as will be seen by the example which follows.

In the case that $\mathcal{B}(., .)$ is a linear time-invariant operator, that is a matrix $\mathcal{B}$, then it gets transformed to $P\otimes Q$ by the above process. It should also be noted that $P$ and $Q$ can be found to bring $\mathcal{B}$ to the form $I + 0$ where $+$ denotes the direct sum and $0$ denotes a zero matrix; in some cases this form for the transformed $\mathcal{B}$ may be desirable but sometimes not.

3. Differentiator example

In order to test out the ideas we tried them on what one might consider as a worst case example, that of a differentiator. This is a linear example for