A POSSIBLE ESTIMATE OF THE ELEMENTARY LENGTH IN ELECTROMAGNETIC INTERACTIONS

J. KVASNICA
Faculty of Technical and Nuclear Physics, Prague

The difference of 0.2 MHz between the theoretical and experimental values of the Lamb shift of the hydrogen terms $2s_{1/2}$ and $2p_{1/2}$ is used to estimate the elementary length $\lambda$ in an electromagnetic interaction. The calculated difference of the hydrogen terms in the Bopp-Podolsky potential field $\varphi(r) = \frac{e}{4\pi r} (1 - e^{-r/\lambda})$ is compared with the given difference $\Delta \approx 0.2$ MHz. From this we then get for the elementary length $\lambda \approx 0.1 \times 10^{-13}$ cm. The distribution of the electric charge in the proton $\varphi(r) = \frac{e}{4\pi\lambda^2} \frac{e^{r/\lambda}}{r}$ leads to the Bopp-Podolsky potential if the Coulomb interaction is assumed to be valid in the whole space.

INTRODUCTION

As is known, in Dirac's theory the electron terms $2s_{1/2}$ and $2p_{1/2}$ (in the Coulomb field of a nucleus) have the same energy. Vacuum corrections cause the splitting of these levels which leads to the known Lamb shift. The theoretical value of the Lamb shift, calculated in the fourth approximation on the assumption of Coulomb interaction between an electron and proton, differs by 0.2 MHz from the experimental value of Lamb's shift [1,2]. This difference may be caused by the finite dimensions of the proton, the invalidity of the usual electrodynamics at small distances, or by a combination of both effects [3].

In the present paper we shall try to estimate the size of the region for which the Coulomb law could be disturbed on the assumption that this disturbance is described by the Bopp-Podolsky potential

$$(1) \quad \varphi(r) = \frac{e}{4\pi r} (1 - e^{-r/\lambda}).$$

This exchange corresponds to the replacement of the usual photon propagator $-1/k^2$ by the Pauli-Villars propagator

$$(2) \quad -\frac{1}{k^2} \sim -\frac{1}{k^2} + \frac{1}{k^2 + \frac{1}{\lambda^2}}.$$
As is known [4], the distribution of the electric charge of the proton

\[ \psi(r) = \frac{e}{4\pi\lambda^2} e^{-r/\lambda} \]

leads to the potential (1) assuming the validity of the Coulomb law in the whole space. The distribution (3) of the proton charge has the mean square radius

\[ \langle r^2 \rangle^{1/2} = \frac{1}{\sqrt{\pi}} \int r^2 \psi^2(r) \, dr \]

The same potential is obtained by solving the Bopp-Podolsky equation [5]

\[ \nabla^2 \left( 1 - \frac{\lambda^2}{\nabla^2} \right) \psi = -\frac{e}{r} \]

for a point distribution of the proton charge. The quantity \( \lambda \) in the Bopp-Podolsky theory has the meaning of a region in which non-local interaction is of fundamental importance. Since the charge distribution in the form of (1) leads on the assumption of Coulomb interaction to the same effects as the modification of the Coulomb potential at small distances (4) for point charges, no distinction can in general be made between these two possibilities.

This connection can, however, be used to calculate the shift of the electron levels. The potential

\[ \delta \psi(r) = -\frac{e}{4\pi r} e^{-r/\lambda} \]

can be regarded as a disturbance of the Coulomb potential and the change \( \delta \varepsilon \) in the energy as a result of this disturbance can be calculated in the known way

\[ \delta \varepsilon = -e \int \psi^* \delta \psi \cdot \psi \, dr \]

where \( \psi \) are eigenfunctions of the electron in the Coulomb field.

**Calculation of Correction to Lamb Shift of Terms 2s\(1/2\) and 2p\(1/2\)**

The eigenfunctions of a hydrogen-like atom have the known form (see e.g. [6]) and for the sake of brevity will not be written out here. Using these functions, for the difference in levels

\[ \Delta = \epsilon_{2s1} - \epsilon_{2s0} = -e \int \delta \psi(r) \cdot \left( \psi_{2s1}^* \psi_{2s1} - \psi_{2s0}^* \psi_{2s0} \right) \, dr \]

caused by the potential (6) we can derive the relation \(^1\)

\[ \Delta = \frac{C^2 Z^2}{N^2 - 1} \left( \frac{Na_0}{2Z} \right)^3 \int_0^\infty \rho^{-1} \exp \left[ -\left( 1 + \frac{Na_0}{2Z^2} \right) \rho \right] \cdot R(\rho) \, d\rho \]

where \( a_0 = 1/\alpha \) is the Bohr radius, \( \alpha = e^2/4\pi \) is the constant of fine structure,

\[ \gamma = \sqrt{1 - Z^2 \alpha^2}, \quad N = \sqrt{2(1 + \gamma)} \]

\[ C^2 = \frac{2\gamma + 1}{T(2\gamma + 1)} \frac{1}{4N} \left( \frac{2Z}{Na_0} \right)^3 \]

\(^1\) Unless otherwise stated, we shall use the system of units \( h = c = 1 \) and Heaviside units for electromagnetic quantities.