Dimensionally Renormalized Green's Functions for Theories with Massless Particles. I.

P. Breitenlohner and D. Maison*
Max-Planck-Institut für Physik und Astrophysik, D-8000 München 40, Federal Republic of Germany

Abstract. In the framework of dimensional renormalization the existence of Green's functions to all orders of perturbation theory is proved for theories of massless particles without super-renormalizable couplings. For those Green's functions Schwinger's Action Principle holds as in the massive case.

I. Introduction

In a previous publication [1], an attempt was made to give a consistent formulation of the so-called "Dimensional Renormalization" to all orders of perturbation theory such that Schwinger's Action Principle holds. That was done under the provision that all particles were massive. In the present paper we want to relax this condition. More precisely we shall treat here only the case that all particles are massless and the theory contains no interactions of super-renormalizable type. In a subsequent publication we shall come to the general case of both massive and massless particles, which is complicated by the fact that additional finite subtractions for subgraphs with positive superficial degree of divergence have to be made in order to guarantee their correct normalization.

In contrast to the BPHZ method the Action Principle holds unmodified by radiative corrections in almost all cases discussed in the literature, with the exception of the few occasions where these corrections are known to be unavoidable (e.g. Trace identities, Adler-anomaly)\(^1\).

The physical relevance of a renormalization scheme that allows the treatment of massless particles on one hand and in which the Action Principle holds on the other is obvious, especially in view of theories with gauge invariance of the second kind. We want to emphasize, however, that we do not tackle the physical infrared problem here, i.e. the definition of the S-matrix for massless particles.

* Present address: II. Institut für Theoretische Physik der Universität Hamburg, Luruper Chaussee 149, D-2000 Hamburg 50, Federal Republic of Germany

\(^1\) The discussion of super-symmetries is still missing in this framework
Dimensional renormalization as outlined in [1] proceeds in two steps:

i) Dimensionally regularized Feynman amplitudes are defined by treating the parameter $n$ of space-time dimension as a regularizing device; a unique prescription is given to extract the $n$-dependence of Lorentz covariants like spin polynomials etc.; these regularized Feynman amplitudes turn out to be distributions which are meromorphic functions of $n$.

ii) The poles of these meromorphic functions are eliminated in a way consistent with additive renormalization.

The regularized Feynman amplitudes are defined via their Feynman parameter integral representation:

$$
\mathcal{F}_G(p,n) = c_G \delta(\sum p_i) \lim_{\varepsilon \to 0} \int_0^\infty dx_G \, I_G(p, z, n)
$$

where $I_G(p, z, n)$ is a distribution in $p$, depending parametrically on $z$ and $n$.

The UV divergencies turn up as non-integrable singularities of the integrand when certain subsets of $\alpha$'s vanish. The nature of these singularities can be displayed by a subdivision of the domain of integration and the introduction of suitable "scaling variables" in place of the $\alpha$'s [2].

In the case of massless particles the integral may also diverge when certain $\alpha$'s tend to infinity. These IR singularities can be analyzed by the same device [3].

Once this has been established it is just a matter of power counting to show that in the absence of super-renormalizable interactions no such singularities of the infrared type are encountered in a neighbourhood of $n=4$ as long as $\mathcal{F}_G(p, n)$ is considered as a distribution in $p$ (i.e. there are clearly physical region singularities, like in the massive case).

The subsequent subtraction of the UV poles in $n$, i.e. the renormalization proceeds like in the massive case [1]. Also in the proof of the Action Principle there is no change.

II. Dimensionally Regularized Feynman Amplitudes

II.1. Analysis of the Singularity Structure

As in the massive case, treated in [1], we start from the formal Feynman parameter integral representation for the amplitude corresponding to some connected graph $G$ with $h_G$ loops

$$
i\mathcal{F}_G(p_1, \ldots, p_M, n) = (2\pi)^{n/2} \delta\left(\sum_{i=1}^M p_i\right) (ih)^{h_G - 1} (2i)^{-n/2} h_G
$$

where

$$I_{G,\varepsilon}(p, y, z, n) = d(\varepsilon)^{-\frac{n}{2}} \int_{\mathcal{F}_G} \, \exp\left[iV(p, y, z) - \varepsilon \sum \mathcal{G}_\varepsilon\right],
$$

the quadratic form $V$ is given by

$$V(p, y, z) = (p^+, y^+) \begin{pmatrix} 0 & -2e_E^+ \\ -2e_E & -4z \end{pmatrix}^{-1} \begin{pmatrix} p \\ y \end{pmatrix}
$$

and $p = (p_1, \ldots, p_{e_G}, \ldots, p_M)$ (cf. [1]).