Commun. math. Phys. 54, 1—19 (1977)

Communications in
Mathematical
Physics

The Back Reaction Effect in Particle Creation in Curved Spacetime*

Robert M. Wald**

Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA

Abstract. The problem of determining the changes in the gravitational field caused by particle creation is investigated in the context of the semiclassical approximation, where the gravitational field (i.e., spacetime geometry) is treated classically and an effective stress energy is assigned to the created particles which acts as a source of the gravitational field. An axiomatic approach is taken. We list five conditions which the renormalized stress-energy operator $T_{\mu\nu}$ should satisfy in order to give a reasonable semiclassical theory. It is proven that these conditions uniquely determine $T_{\mu\nu}$, i.e. there is at most one renormalized stress-energy operator which satisfies all the conditions. We investigate existence by examining an explicit "point-splitting" type prescription for renormalizing $T_{\mu\nu}$. Modulo some standard assumptions which are made in defining the prescription for $T_{\mu\nu}$, it is shown that this prescription satisfies at least four of the five axioms.

I. Introduction

In the past several years, a considerable amount of progress has been made in our understanding of quantum processes occurring in a strong gravitational field. A satisfactory quantum theory of the gravitational field itself still does not exist [1]. However, the framework of a semiclassical theory describing other quantum fields present in a strong gravitational field does exist and has been used to investigate particle creation effects. In this theory the gravitational field is described in an entirely classical manner as curvature in the geometry of spacetime, in accordance with the notions of general relativity. The fields (e.g., a scalar, Dirac, or Maxwell field) which are present in spacetime are described in accordance with the principles of quantum field theory. It is not believed that this theoretical framework can provide an exact description of nature, since it cannot be entirely consistent to have

* Supported in part by the National Science Foundation under grant MPS 74-17456-A01 and by the Sloan Foundation

** Sloan Foundation fellow

quantum fields (described in probabilistic terms) interact with a classical gravitational field (with definite, determined values). Rather, this semiclassical theory is viewed as an approximation to the true—as yet unknown—quantum theory of gravitation interacting with other fields. Such a semiclassical framework is analogous to the situation in atomic physics where, for the description of a wide range of phenomena, it is a good approximation to describe the electromagnetic field in an entirely classical manner while treating the electrons quantum mechanically. On dimensional grounds it is generally believed that quantum effects of gravity should be important at least when the spacetime curvature becomes comparable to the Planck length $(\hbar G/c^3)^{1/2} \approx 10^{-33}$ cm. However, for less extreme spacetime curvature, one hopes that the semiclassical approximation will be valid at least in many situations.

If the gravitational field has suitable asymptotic behavior in the past and future, a description of the quantum fields in terms of particles will be possible in these asymptotic regimes. One may then ask about particle creation: If the field is initially in the vacuum state, how many particles will be present at late times? More generally, what is the S-matrix? It turns out that a few simple assumptions within the semiclassical framework described above uniquely lead to an expression for the S-matrix in a manner which is very nearly free of any mathematical difficulties [2]. Thus, one can make well defined, unambiguous predictions concerning particle creation in a strong gravitational field.

The most remarkable application of these ideas is, of course, Hawking's discovery [3] that particle creation near a Schwarzschild black hole will result in a steady rate of emission of particles with an *exactly* thermal spectrum [2,4]. This result is particularly striking in view of the analogies that had previously been discovered between black hole physics and thermodynamics [5,6]. In the absence of any experimental or observational confirmation of the predictions of the semiclassical theory, it is the beauty of Hawking's result as well as the simplicity, naturalness, and good mathematical behavior of the theory which gives one confidence that this approach is on the right track.

In the particle creation calculations referred to above, the spacetime geometry (i.e., gravitational field) is taken to be that appropriate to some classical physical situation, e.g., the gravitational collapse of a body to form a black hole. The particle creation is then calculated in this fixed spacetime geometry. However, on physical grounds it is clear that the quantum particle creation must have some "back reaction" effect on the spacetime geometry. In particular, for the case of a black hole, the particle creation calculations show a flux of energy coming from the black hole. By conservation of energy one would expect this energy flux to be balanced by a decrease in the mass of the black hole (i.e., a decrease in the energy of the gravitational field). The determination of the nature and magnitude of the "back reaction" effect is of great interest and importance in its own right, particularly in the cosmological context where the "back reaction" of particle creation may have an important effect on the dynamics of the universe. It is also needed to check the validity of the particle creation calculations, since if the effect of the "back reaction" is large, it must be taken into account in these calculations.

In what framework can one analyze this "back reaction" effect? It is conceivable that one will need a complete quantum theory of gravitation in order to describe it,