DISTURBING FORCES RESPONSIBLE FOR THE ACTUAL FIGURE OF PALLAS

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Summary: The tri-axial figure of the asteroid 2 Pallas poses the question of the origin of this phenomenon. The tidal-forming potential and rotational disturbances may be fully responsible for it. However, this hypothesis requires Pallas to be a primordial satellite orbiting the hypothetical central body synchronously, i.e. in 1/1 orbital/rotational resonance; its mean motion has been estimated.

1. INTRODUCTION

The origin of the asteroids is still an open problem, in general. The available data are inadequate for deriving a dynamically well based solution. However, the parameters of the tri-axial figure, if known, could be a key point for the solution. The crucial question should be posed and answered: Which forces are responsible for the actual triaxial figure of an asteroid?

The three well-determined axes of Pallas [1] suggest that one should examine whether the tidal and centrifugal forces might be responsible for the actual figure of Pallas, and whether the last can be believed to be a primordial satellite synchronously orbiting around a "hypothetical" central (primary) body.

2. TRI-AXIAL FIGURE OF PALLAS AS A RESULT OF THE TIDAL AND ROTATIONAL DISTORTIONS DUE TO HYPOTHETICAL ROTATIONAL/ORBITAL RESONANCE

Let $k_s$ and $k_t$ be the secular Love [2] and tidal (introduced in [3]), numbers, respectively, defining the deforming potential $\delta Q$ due to rotation, and the additional tidal potential $\delta V_t$ in this special case as follows:

$$\delta Q = -\frac{1}{3} k_s \frac{Gm}{\rho} \left( \frac{a_0}{a} \right)^{-3} q P_2^0 (\sin \Phi) ,$$

$$\delta V_t = -\frac{1}{2} k_t \frac{GM}{a} \left( \frac{\rho}{a} \right)^2 \left[ P_2^0 (\sin \Phi) - \frac{1}{2} P_2^2 (\sin \Phi) \cos 2\Lambda \right] ,$$

$$q = \frac{\omega^2 a_0^3}{Gm} ;$$

$Gm$ and $GM$ are the centric gravitational constants of the satellite (S) and of the central body (P), respectively; $\rho$, $\Phi$, $\Lambda$ are the centric spherical coordinates (the radius-vector,

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latitude, longitude, respectively) of a potential point \((N)\) on \(S\); \(a\) is the semi-major axis of the orbit of \(S\) assumed to be situated in the equatorial plane of \(P\); \(\omega\) is the angular velocity of rotation of \(S\); \(a'_0\) is the length scaling parameter chosen here equal to the mean equatorial radius of \(S\); \(P_2^{(0)}(\sin \Phi) = 3/2 \sin^2 \Phi - 1/2, P_2^{(2)}(\sin \Phi) = 3 \cos^2 \Phi\).

Because of synchronous orbital/rotation motion, \(\omega = n\) = mean motion of \(S\), and

\[
\frac{\omega^2 a_0^3}{Gm} = \frac{GM}{Gm} \left( \frac{a'_0}{a} \right)^3 = q. \tag{4}
\]

If potentials (1) and (2) are fully responsible for the actual figure of \(S\), then the second-degree Stokes parameters \(J_2^{(0)}, J_2^{(2)}\) should read

\[
J_2^{(0)} = -\left( \frac{R_0}{a'_0} \right)^5 q \left( \frac{1}{3} k_s + \frac{1}{2} k_t \right), \tag{5}
\]

\[
J_2^{(2)} = \frac{1}{4} \left( \frac{R_0}{a'_0} \right)^5 q k_t ; \tag{6}
\]

\[
R_0 = (a' b' c')^{1/3} \tag{7}
\]

is the mean radius of \(S\), and \(a' > b' > c'\) its semi-axes. However, if \(S\) is believed to be homogeneous, \(J_2^{(0)}\) and \(J_2^{(2)}\) are functions of \(a', b', c'\) only, and we get two conditions:

\[
q k_s = -3 \left( \frac{a'_0}{R_0} \right)^5 \left( J_2^{(0)} + 2 J_2^{(2)} \right) = \frac{3}{5} \left( \frac{a'_0}{R_0} \right)^5 \frac{b^2 - c^2}{a_0^2} , \tag{8}
\]

\[
q k_t = 4 \left( \frac{a'_0}{R_0} \right)^5 J_2^{(2)} = \frac{1}{5} \left( \frac{a'_0}{R_0} \right)^5 \frac{a^2 - b^2}{a_0^2} . \tag{9}
\]

If parameter \(q\) were known, secular numbers \(k_s, k_t\) could be determined, and the rigidity of \(S\) estimated. However, in the opposite case (Pallas) given, we face the "inverse" problem, and \(k_s, k_t\) should be given a priori.

The figure parameters of Pallas were determined by Dunham et al.\[1\] from the 1983 occultation of 1 Vulpeculae as

\[
a' = 287 \text{ km}, \quad b' = 263 \text{ km}, \quad c' = 250 \text{ km}, \tag{10}
\]

as well as its mean density

\[
\bar{\sigma} = 3400 \text{ kg m}^{-3} , \tag{11}
\]

which yields the centric gravitational constant of Pallas as

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