3. CONCLUSION

In the affine transformation with the matrix $A$ the lengths are deformed in dependence on the direction of the transformed vector in space. The extreme values of the ratio of lengths of the corresponding vectors are equal to the square roots of the largest and smallest eigenvalue of matrix $AA^T$, respectively. Provided these numbers do not differ too much, the extreme deformations of the lengths can be approximated by formulae (27) and (28). The use of both formulae only assumes the knowledge of the determinant of the transformation matrix and of the sum of the squares of the elements of this matrix. For the general affine transformation according to Eq. (1b) analogous conclusions hold true. However, instead of the vector $y$ one must of course consider the vector $(y - b)$.

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WEIGHTING OF VERTICAL ANGLES

Dedicated to 90th Birthday of Professor František Fiala

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Summary: According to the results of the adjustments of eight trigonometric and three-dimensional networks, the a priori variance $m^2(\beta)$ of the measured vertical angle $\beta$ is expressed by the formula: $m^2(\beta) = m^2(a) + \left[ C \frac{1}{2} m(k) \right]^2$, where $m(a)$ represents accidental observation errors; the constant $C$ is estimated in the interval 0.5–1.5 according to the number of repeated observations and the variation of their changes with time; $\gamma$ is the angle between the normals to the ellipsoid at the initial and final point of the line of sight, and $m(k)$ is the mean square error of the coefficient of refraction which can be estimated for a given network from Tab. 1.

1. THE RESULTS OF ADJUSTING EIGHT NETWORKS

The values of the unknown parameters, determined by the adjustment of geodetic networks, are usually little affected by the incorrect choice of weights; however, a reliable estimate of the weights is important for determining the mean square errors of these parameters correctly [5, 6]. In adjusting eight trigonometric and three-dimensional networks in mountain regions, as well as in planes [2, 5, 7], the following results were found:

1. A large difference between the a priori mean square error of the vertical angle, derived from accidental observation errors, and the mean square error after adjustment;

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2. The ratio of the a priori errors and those after the adjustment varied according to the mathematical model that was used for expressing of refraction. If a constant coefficient of refraction was assumed for the whole network, this ratio was 1 : 6, if the coefficient of refraction was variable, expressed by a single unknown parameter for each point, the ratio was 1 : 2.2 [2, 7].

3. The distribution of residuals of the vertical angles, determined by the adjustment, also varied according to the expression of refraction. If the coefficient of refraction was constant, expressive systematic errors were in evidence at some of the standpoints, and these vanished after the introduction of a variable coefficient of refraction.

It can thus be seen that for a more reliable expression of the a priori mean square errors and weights of observed vertical angles it is necessary to supplement the accidental observation errors with a component, expressing the uncertainty due to refraction which has a random, as well as systematic character and which depends on the refraction model adopted in adjusting the network\(^1\)). The investigation of the variations of the observed vertical angles with time, determined from repeated measurements, will also be important for determining the uncertainty due to refraction.

2. A PRIORI MEAN SQUARE ERROR OF THE OBSERVED VERTICAL ANGLE

The apriori variance (i.e., the square of the mean square error) \( m^2(\beta) \) of the observed vertical angle \( \beta \), which is indirectly proportional to the weight, will be expressed as the sum of the squares of two components:

\[
(1) \quad m^2(\beta) = m^2(a) + m^2(r) .
\]

The component \( m(a) \) represents accidental observation errors; in measuring with Wild T2, T3, and Zeiss Theo 010 theodolites it amounts to about 2\(^\circ\), assuming that suitable sighting signals and a suitable measuring procedure which will eliminate short-period variations of refraction by repetition of pointing, are used [1, 3].

The component \( m(r) \) is an error due to refraction; it can be expressed in terms of the mean square error \( m(k) \) of the refraction coefficient:

\[
(2) \quad m(r) = \frac{1}{2} \gamma m(k) ,
\]

where \( \gamma \) is the angle between the normals to the ellipsoid at the initial and final points of the line of sight, the magnitude of which depends, in the first place, on the length of the line of sight. The values of \( m(k) \) will be adopted from earlier adjustments (Tab. 1)\(^2\)).

The mean square errors, determined according to Eq. (2), exceed the component \( m(a) \) considerably with lines of sight in excess of 5 km and fully conform to the differences between the a priori mean square error of the observed vertical angle and its error, determined by adjustment, mentioned, above. If one considers all the lines of sight in the network in which a different coefficient of refraction has been determined for each standpoint, the distribution of the residuals of

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1) The systematic component of the refraction error will also contain other systematic errors, e.g., the error due to the division of the circle and the error due to neglecting the higher terms in linearizing the observation equation, or the condition equation. Under normal conditions of observation and observation processing, considering lengths of sides over 5 km, these errors are statistically insignificant.

2) The conditions for determining the refraction and the deflections of the vertical from vertical angles are given in [3, 4]. The most important of these is the measuring of all zenith distances, originating at a standpoint, within a short time interval.