KINEMATICS OF REFRACTED AND REFLECTED WAVES IN INHOMOGENEOUS MEDIA WITH NON-PLANAR INTERFACES

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1. INTRODUCTION

Recently, attention is being turned to problems of propagation of elastic waves in inhomogeneous media, in which the velocity depends on two or three co-ordinates. This is the result of the endeavour to model as accurately as possible real media in which elastic waves propagate.

The problem is in solving the equation of motion, in which Lamé's elastic parameters $\lambda$ and $\mu$ and the density of the medium $\rho$ and, therefore, also the velocity of the elastic waves, are in general a function of the position in the medium. The asymptotic (ray) solution of these equations in the form of a ray series is described in the fundamental papers [1 -- 3]. The procedure, described in the said papers, can be used to derive the eiconal equation from the equation of motion; the solution of the former, a scalar time field as a function of co-ordinates, represents the solution of the kinematic part of the problem.

The formal solution of the eiconal equation, however, can only be found in certain special cases. In the present paper, therefore, this equation has been transformed into a system of ordinary differential equations of the first order, which can be solved numerically. For example, there is a system of this kind in [4, 5]. In [5] these equations have been used to solve the problem of propagation of acoustic waves.

The problem may also be generalized by introducing non-planar interfaces of the first order, at which the elastic parameters, and, therefore, also the velocities, change discontinuously, e.g., in [6, 7]. Moreover, [7] also gives a possible algorithm of the inverse problem, i.e., determining the interface from the travel-time curves of the reflected waves.

By applying the procedures given in [1 -- 3] a system of ordinary differential equations of the first order can be constructed, similar to the equations for kinematics, and the amplitudes can be computed [8]. A possible way of determining the velocity distribution in an inhomogeneous medium from the travel-time curves of refracted waves is given in [9].

The present paper deals with the problems of the kinematics of propagation of seismic waves in inhomogeneous, isotropic media with non-planar interfaces. The fundamental relations for

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computing the rays of seismic waves in curvilinear orthogonal co-ordinates have been derived for cases in which the velocity of the seismic waves depends on two co-ordinates, as well as on three. Finally, these relations have also been supplemented by formulae for computing rays refracted and reflected at non-planar interfaces. The treatment of these problems required a slightly different approach than the treatment of problems, in which the velocity only depends on two co-ordinates (compare [6, 7]).

Computer programmes were written for selected alternatives of the problems mentioned below. Examples are given of computing the kinematics of reflected and refracted waves for given models of the Earth’s crust, which were found by deep seismic sounding along profile VI in Czechoslovakia. The velocity observed in the crust depends on the depth as well as the epicentral distance. A larger set of similar computations may serve to check the results obtained by approximate methods which do not take into account the horizontal gradient of the velocity, or only consider it in the form of corrections.

2. THEORETICAL RELATIONS

The theoretical problem of the propagation of elastic waves in inhomogeneous, perfectly elastic and isotropic media consists in the solution of the linearized equation of motion for an inhomogeneous medium:

\[
q \frac{\partial^2 W}{\partial t^2} = (\lambda + \mu) \nabla (\nabla \cdot W) + \mu \nabla^2 W + \nabla \lambda (\nabla \cdot W) + \nabla \mu \times (\nabla \times W) + 2(\nabla \mu \cdot \nabla) W,
\]

in which the density of the medium \( q \) and Lamé’s elastic parameters \( \lambda \) and \( \mu \) are continuous functions of spatial co-ordinates. The displacement vector \( W \) is also a function of the co-ordinates and of time as well. The equation of motion in its vectorial form (1) is invariant with respect to the system of co-ordinates; therefore Eq. (1) can be solved in any system of orthogonal curvilinear co-ordinates \( q_1, q_2, q_3 \). Thus, \( q = q(q_1, q_2, q_3), \lambda = \lambda(q_1, q_2, q_3), \mu = \mu(q_1, q_2, q_3) \) and \( W = W(q_1, q_2, q_3) \). In order to find the solution to Eq. (1), the following procedure will be applied: The solution will be sought in the form of a ray series

\[
W(q_1, q_2, q_3, t) = \exp \{i\omega [t - \tau(q_1, q_2, q_3)] \} \sum_{n=0}^{\infty} (i\omega)^{-n} W^n(q_1, q_2, q_3),
\]

where \( \omega \) is the frequency of a harmonic disturbance, \( \tau \) is the phase function (\( \tau(q_1, q_2, q_3) = t \) is the equation of the wave front), and \( W^n \) are the coefficients of the ray series. \( W^n \) and \( \tau \) depend neither on \( t \), nor on \( \omega \). These quantities substituted into Eq. (2), yield the ray solution of Eq. (1).

In order to determine \( \tau \) and \( W^n \), Eq. (2) is substituted into (1), which yields

\[
\sum_{n=0}^{\infty} (i\omega)^{-n} [(i\omega)^2 A(W^n) - (i\omega) B(W^n) + C(W^n)] = 0,
\]

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