UNDERSTANDING NATURAL FREQUENCY AND DAMPING AND HOW THEY RELATE TO THE MEASUREMENT OF BLOOD PRESSURE

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ABSTRACT. The model that describes the physical behavior of a fluid-filled catheter–transducer blood pressure monitoring system is a simple mass-spring system. When the mass is displaced and then released, there results a characteristic motion called simple harmonic motion. The full description of this motion requires defining the concepts of undamped and damped natural frequency, as well as of damping itself. Once these concepts are defined and the mass-spring system clearly understood, their relevance to recording blood pressure measurement by fluid-filled catheters is explained. The apparent paradox of how damping can affect undamped natural frequency is clarified. Finally, impedance matching is explained in the context of how some damping devices work. Detailed mathematical proofs are relegated to an appendix.


Fluid-filled catheters attached to transducers are commonly used to measure intraarterial blood pressure. To intelligently assess the data generated by such devices, one must understand certain basic concepts: (1) undamped natural frequency; (2) damped natural frequency; (3) damping; and (4) resonant frequency. To help understand these concepts, it is useful to understand the model on which the descriptive behavior of a fluid-filled catheter transducer system is based. The components of a mass–spring system include inertial forces represented by the mass, elastic forces represented by the spring, and friction forces represented by the medium that the mass–spring is in. If this model is applied to a fluid-filled catheter recording system, the inertial (mass) forces are represented by the fluid in the catheter, the elastic forces by the transducer diaphragm, and the friction forces by the viscous effects of the moving fluid in the catheter. For practical purposes, therefore, the inertial and friction forces are confined to the fluid in the catheter and the elastic forces to the transducer diaphragm. In addition, for simplicity, no distinction will be made between the catheter and the pressure connecting tubing since they both behave qualitatively alike.

A fluid-filled catheter recording system incorporating a flush device will allow the physician to perform a square-wave test (Fig 1). This test exposes the recording system to a sudden pressure change, such that a sinusoidal pressure wave of a given frequency and progressively decreasing amplitude is recorded at the transducer. This is the recorded motion that would be seen if one were to displace a mass attached to a spring from its equilibrium position and release it (Fig 2). The behavior...
Fig 1. (A) The input signal to a blood pressure recording system: square wave depicting a sudden pressure change. (B) The output signal: the use of a flush device to produce a square wave on a fluid-filled catheter generates a sinusoidal pressure wave of progressively decreasing amplitude and constant frequency.

of this model—the mass and spring system—is analogous to the behavior of a fluid-filled catheter.

ANALYSIS OF MASS-SPRING SYSTEM

Harmonic Motion

If the mass is displaced from its equilibrium position and released, its motion could be described as harmonic. To

Fig 2. (A) Spring with resting length L. (B) Mass-spring system. Mass (M) stretches the spring by length \( \ell \). Its equilibrium position is now at \( L + \ell \). The vertical lines on each side of the mass represent the friction or viscous component of the system. (C) Mass is pulled down a distance \( x \) and then released. Resulting motion is recorded (damped harmonic motion). Arrow points in direction of motion of recording paper (abscissa is time).

fully understand harmonic motion, it is necessary to understand the concept of angular or circular frequency \( (\omega_o) \). Harmonic motion can be thought of as the projection on a straight line \( (Z) \) of a point \( (P) \) that is moving on a circle at constant speed (see Fig 3) [1]. If the angular speed of the line \( OP \) in Figure 3 is designated by \( \omega_o \), the displacement \( x \) can be shown as

\[
    x = A \sin \omega_o t, \tag{1}
\]

where \( t = \) anytime and \( \omega_o \) is in radians/s \((2\pi \text{ radians} = 360^\circ)\). The motion repeats every \( 2\pi \) radians. Therefore,

\[
    \omega_o = 2\pi/T,
\]

where \( T \) (the period) = time to complete one full oscillation). If \( fn \) stands for the undamped natural frequency (in hertz) which is the reciprocal of the period \( 1/T \), then,

\[
    \omega_o = 2\pi fn
\]

Therefore \( fn = \omega_o/2\pi \). Equation 1 describes a system of one degree of freedom. The motion is completely specified by the determination of how one variable (\( x \)) varies with time [2]. The motion of the mass is ultimately determined by three factors: (1) mass; (2) stiffness of the spring; and (3) friction. The number of oscillations per unit time of the mass is known as the frequency. Because these oscillations occur in the real world, in the presence of friction, this frequency is known as the damped natural frequency. The frequency of oscillations occurring in the absence of frictional forces is known as the undamped natural frequency [3]. The undamped natural frequency therefore is an idealized situa-