LOADING WITH THE CURRENT OF A LINEAR ACCELERATOR BUNCHER

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The main problem in calculating a waveguide for a linear accelerator buncher is finding the dependence of the geometry of the waveguide diaphragms on the length. If we neglect all forms of losses of high-frequency power, then this problem amounts to finding the law of change in dimensions of the diaphragms along the waveguide based on conditions for obtaining maximum bunching of the particles.

The dimensions of the diaphragms of a waveguide are determined from a dispersion equation which connects the phase velocity of a wave $v_f$ with a frequency $\omega$ and the dimensions of the waveguide

$$F(\gamma, a) = f(\omega, a),$$

where $a$ is the radius of the hole in the waveguide diaphragms; $\gamma = \sqrt{\left(\frac{\omega}{v_f}\right)^2 - \left(\frac{\omega}{c}\right)^2}$ is the natural value of the problem of propagation in a waveguide of a wave type $E$ ($c$ is the velocity of light). Change in $\gamma$ from $z$ ($z$ is the longitudinal coordinate of the waveguide) can be selected arbitrarily.

It is essential to allow for the effect of a large current on the decrease in amplitude of the traveling wave. In accordance with this the values of $\gamma$, and hence the radii of the holes in the waveguide diaphragms, will be functions of the loading of the waveguide by the current of particles. In [1, 2] a study was made of the effect of the current of the beam on the change in amplitude of the accelerating field $E_z$ in the waveguide with a constant phase velocity. The method of solution used cannot calculate the geometry of the buncher. A method is considered below for obtaining an approximate dependence of the value $\gamma$ on $z$ for a waveguide with a variable phase velocity with an allowance for its loading by the current of the beam and power losses in the walls. Knowing the dependence of $\gamma$ on $z$ we can readily determine the required length of waveguide from given values of the final energy, current of particles, power of the generator, accelerating voltage and losses in the waveguide walls.

The equation for the balance of high-frequency power current in any cross section of an accelerating waveguide has the form [3]

$$\frac{dP}{dz} = -IE_z \sin \varphi - 2\alpha P,$$

where $P$ is the high-frequency power current causing a field $E_z$; $I$ is the current of a beam of particles; $\varphi$ is the equilibrium phase of the particle; $\alpha$ is the attenuation in the waveguide, caused by losses in the walls. The solution of Eq. (2) can be written in the form

$$P = \left(P_0 - I \int e^{2\alpha z} dU\right) e^{-2\alpha z},$$

where $dU = E_z \sin \varphi dz$; $P = P_0$ when $z = 0$; $U$ is the kinetic energy of the particle. The dependence of the function $U$ on $z$ is unknown and hence the integral (3) cannot be calculated. It can be represented in the form

$$\int (1 + 2\alpha z + 2\alpha^2 z^2) dU$$
with the condition of sufficient smallness of \( \alpha z \), which is very often the case in practice. Equation (4) can be written thus:

\[
U + \int (2\alpha z + 2\alpha^2 z^2) dU = U \left( 1 + \varrho \right),
\]

where

\[
\varrho = \frac{1}{U} \int (2\alpha z + 2\alpha^2 z^2) dU.
\]

Expression (3) is represented in the following way:

\[
P = [P_0 - IU(1 + \varrho)] e^{-2\alpha z}
\]

Equation (5) is an approximate solution of Eq. (1) with the assumption of smallness of \( \alpha z \). The method of the approximate determination of \( \rho \) is given below.

The high-frequency power current in the waveguide is expressed by the formula

\[
P = \tau E_z^2 \Gamma,
\]

where \( \tau \) is a constant; \( \Gamma \) is a function depending on the radius of the hole in the waveguide diaphragms and the parameter \( \gamma \).

Comparing Eqs. (5) and (6) we can obtain an expression for \( E_z \):

\[
E_z = e^{-\alpha z} \sqrt{\frac{P_0 - IU(1 + \varrho)}{\tau \Gamma}}
\]

or

\[
\frac{1}{\sin \varphi} \frac{dU}{dz} = e^{-\alpha z} \sqrt{\frac{P_0 - IU(1 + \varrho)}{\tau \Gamma}}.
\]

In Eq. (8) \( \sin \varphi \) can be given or at least we can know the function \( z \).

Since [see (1)]

\[
\gamma = k \sqrt{\left( \frac{c^2}{v_f} \right)^2 - 1 - \delta}
\]

\( k = \frac{\omega}{c} \); \( \delta \) is a parameter allowing for the effect of the current of particles on the dispersion properties of the waveguides at currents up to 0.5 a, \( \delta = 1 \) and for a particle moving at a velocity \( v \)

\[
\frac{v}{c} = \sqrt{1 - \left( \frac{U_0}{U_0 + U} \right)^2}
\]

\( U_0 \) is the rest energy of the particles, then the derivative of \( U \) with respect to \( \gamma \) is equal to

\[
\frac{dU}{d\gamma} = -U_0 \frac{k^2}{\gamma^2} \frac{1}{\sqrt{1 + k^2 \gamma^2}}
\]

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