STRESSES AND DISPLACEMENTS DUE TO A STATIONARY POINT SOURCE OF HEAT IN AN ELASTIC HALFSpace

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1. INTRODUCTION

The theory of thermo-elastic phenomena, presented, e.g., in [1—4], indicates that if an inhomogeneous temperature field exists in an elastic continuum, thermo-elastic stresses and deformations may arise in it. Experiments and everyday experience have proved that, as a result of these stresses, many bodies become destructed, because cracks and fractures are produced. Geothermal measurements have shown that the temperature field of the Earth is considerably inhomogeneous not only vertically, but also horizontally. We may, therefore, expect various thermo-elastic phenomena to occur also in some regions with an anomalous behaviour of the heat flow.

The thermo-elastic stresses and deformations of the Earth as a whole, caused just by the variation of temperature with depth are investigated in [5]. In this paper we shall concentrate in analysing thermo-elastic stresses and displacements due to expressive inhomogenities of the temperature field in the Earth of smaller horizontal and vertical dimensions — about one hundred kilometres. It was indeed found [6, 7] that in areas where considerable horizontal changes in the value of the heat flux were observed, seismic activity was increased in the surface layers of the Earth's crust, i.e. that an accumulation and release of stresses occurred. Using a theoretical model we shall present quantitative relations between the inhomogeneous temperature field due to a point heat source and the stress, which can arise as a result of thermo-elasticity. We shall prove that tensile stresses, amounting to as much as the limit of strength of rocks, can be generated at the surface of the halfspace, and that this is capable of providing a considerable amount of geodynamic effects in the neighbourhood of geothermal anomalies.

We shall approximate the lithosphere of the Earth by a homogeneous elastic halfspace, density $\rho$, with the Lamé elastic parameters $\lambda$, $\mu$ and the thermal conductivity $\lambda_0$. We do not consider the thermo-elastic phenomena due to the large-scale temperature field of the Earth, because we assume it to vary with depth only and, therefore, its effects are independent of the horizontal co-ordinates. In order to investigate thermo-elastic phenomena, due to heat anomalies of small horizontal dimensions, we shall adopt a point heat source in the halfspace as the source of the temperature
field. Although this represents a considerable idealization of the actual conditions within the Earth, the solution we shall obtain will yield valuable information with regard to understanding the associations among some of the phenomena observed in geophysics and geodynamics.

2. STATIONARY POINT HEAT SOURCE IN AN ELASTIC HALFSPACE

In an elastic halfspace, \( z \geq 0 \), at a point with the co-ordinates \( x = 0, y = 0, z = \zeta \), we assume a point heat source, with rate of heat production \( w \) [W]. Assume the thermal conductivity of the halfspace to be \( \lambda_0 \). Also assume the surface of the halfspace to be kept at a constant temperature which we shall assume to correspond to the zero of our temperature scale. The stationary temperature field satisfies Poisson’s equation

\[
\nabla^2 T = -w\lambda_0^{-1} \delta(x) \delta(y) \delta(z - \zeta),
\]

with the boundary condition \( T(x, y, 0)|_{z=0} = 0 \) (\( \delta(t) \) is Dirac’s function). The analytical expression of this temperature field is known, e.g., from [8]:

\[
T(x, y, z) = w(4\pi\lambda_0)^{-1} \left[ R_1^{-1} - R_2^{-1} \right],
\]

where \( R_1 = \left[ x^2 + y^2 + (z - \zeta)^2 \right]^{1/2} \) and \( R_2 = \left[ x^2 + y^2 + (z + \zeta)^2 \right]^{1/2} \). The inhomogeneous temperature field (2) generates thermo-elastic stresses and deformations. The stress has been calculated in [1]; we shall only give it in a condensed form, supplementing it with a calculation of the displacement and the corresponding numerical computations.

According to [1], for a field of displacements \( u \), generated by an inhomogeneous temperature field \( T \) in an elastic continuum,

\[
(\lambda + \mu) \text{grad div } u + \mu \nabla^2 u - \gamma \text{ grad } T = 0,
\]

in Cartesian co-ordinates, where \( \gamma = (3\lambda + 2\mu) \alpha, \alpha \) is the thermal coefficient of linear expansion. We can see that the term \( -\gamma \text{ grad } T \) plays the role of a body force in the equation of elastic equilibrium. We are not considering the effect of gravity as another important body force in Eq. (3), because it is assumed to be independent of the horizontal co-ordinates, however, we shall take it into account in analysing the stress.

The solution of Eq. (3) for the temperature field (2) can be conveniently expressed in cylindrical co-ordinates \( (\rho, \varphi, z) \) with the polar axis \( z \); the physical analysis indicates that all physical quantities will be independent of the azimuth \( \varphi \) and the displacement \( u_\varphi \) will be zero. The non-zero components of the displacement are \( u_\rho \) and \( u_z \). According to the theory of elasticity the components of the tensor of deformation \( \varepsilon_{ij} \) will read

\[
\varepsilon_{rr} = \partial u_\rho/\partial r, \quad \varepsilon_{\rho\varphi} = u_\rho/r, \quad \varepsilon_{zz} = \partial u_z/\partial z, \\
\varepsilon_{r\varphi} = 0, \quad \varepsilon_{\varphi z} = 0, \quad \varepsilon_{zz} = 1/2(\partial u_\rho/\partial z + \partial u_z/\partial r).
\]