PARTIAL DERIVATIVES OF TRAVEL-TIME CURVES OF REFLECTED WAVES IN A LAYERED MEDIUM

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Резюме: Выведены простые формулы для расчета частных производных годографов отраженных волн по параметрам слоистой среды.

1. INTRODUCTION

In interpreting travel-time curves it is very valuable if one knows how the travel-time curve varies under small changes of the parameters of the medium. These changes can be characterized by means of the partial derivatives of the travel-time curves with respect to the parameters of the medium. Besides this, the calculation of the partial derivatives is a necessary constituent of some methods of solving the inverse problem.

The partial derivatives of the travel-time curves can be determined by numerical differentiation. However, this method is time consuming and not very accurate, because, in the case of a layered medium, the travel-time curve cannot be expressed by a single formula, but only in so-called parametric form. It is much more convenient to calculate the derivatives of travel-time curves of a reflected wave by means of Eqs (8), given below.

Section 2 gives the well-known formulae for the travel-time curve of a reflected wave. The formulae for the partial derivatives of travel-time curves with respect to the parameters of the medium are given in Section 3 and the way in which they were derived is described in Section 4. Some of the properties of these partial derivatives are described in Section 5 and in Section 6 a numerical example is presented.

Equations (8) for calculating the partial derivatives of travel-time curves of reflected waves were already published in [3], and the use of these partial derivatives for interpretations was described in [5, 6]. However, it took a long time before these formulae were derived accurately. The proof in [3] was founded on the use of Taylor’s developments (ref. to Proof 2 below), the proof in [6] was already founded on differentiating the parametric form of the travel-time curve. However, both proofs contained some steps which were not quite accurate. A new contribution of this paper is Proof 3 which is based on the parametric form (2). It is only this proof that can be considered as accurate from a mathematical point of view. Besides this, also some results from papers [3, 5, 6] are given, because these papers were not published in journals.

2. MODEL OF THE MEDIUM AND THE TRAVEL-TIME CURVE OF A REFLECTED WAVE

We shall consider a medium composed of \( n \) plane-parallel, homogeneous and isotropic layers, lying on a substratum (Fig. 1). Assume that the source of the seismic waves and the observer are located on the surface of the medium. Assume the

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propagation of seismic waves which were reflected from the interface between the $n$-th layer and the substratum. Also assume that multiple reflections and transformation of waves do not occur (i.e. the wave propagates along the whole ray either as a $P$-wave or as an $S$-wave).

![Model of the medium and rays of the reflected wave.](image)

The equation of the travel-time curve expresses the dependence of the travel time $t$ on the epicentral distance $r$ and on the parameters of the medium:

$$ t = t(r, v_1, v_2, \ldots, v_n, d_1, d_2, \ldots, d_n), $$

where $v_m$ and $d_m$ are the velocity of the seismic waves and the thickness of the $m$-th layer, respectively. Apart from the simplest case of one layer ($n = 1$), Eq. (1) cannot be expressed explicitly. Therefore, the equation of the travel-time curve is usually written in parametric form

$$ (2a, b) \quad t = \tau(v_1, \ldots, v_n, d_1, \ldots, d_n, p), \quad r = \xi(v_1, \ldots, v_n, d_1, \ldots, d_n, p), $$

where

$$ p = \sin i_1/v_1 = \sin i_m/v_m $$

is the parameter of the ray and $i_m$ is the angle of incidence in the $m$-th layer. Therefore,

$$ \sin i_m = v_mp = v_m(\sin i_1)/v_1. $$

We have introduced two symbols for time, $t$ and $\tau$, because we consider both quantities to be functions of different variables. Similarly, for the epicentral distance we use the symbols $r$ and $\xi$; $r$ is considered to be the independent variable, whereas $\xi$ is a function of the parameters of the medium and of the parameter of the ray.

It follows from Fig. 1 that the r.h.s. of the equation of the travel-time curve (2) can be altered to read

$$ (5a, b) \quad \tau = 2 \sum_{m=1}^{n} \tau_m, \quad \xi = 2 \sum_{m=1}^{n} \xi_m, $$

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