The proof is based on the following lemma.

**LEMMA.** If \( t = (x, s) \), then \( \tilde{t} = (x, \tilde{s}) \). The proof is by induction on the number of conversions in normalizing \( t \) based on the fact that from \( t = (x, s) \rightarrow t' \) follows \( t' = (x, s') \) and \( s \rightarrow s' \).

The proof of Theorem 2 is concluded.

We recall that the sequent \( S \) is called balanced if each propositional variable occurs in it no more than twice.

Remark. If \( \alpha \) is an arbitrary derivation of the balanced sequent \( S \), and \( \alpha' \) is the derivation of the \( R \)-sequent \( S' \), into which \( \alpha \) is transformed, then \( S' \) is also balanced. In fact, it suffices to note that upon passage from \( S \) to \( \theta(S) \) or \( S\tilde{\varphi} \) (and also, of course, permutation of terms of a conjunction) balancedness is preserved.

**LITERATURE CITED**


**REPRESENTATION OF PROOFS BY COLORED GRAPHS AND THE HADWIGER CONJECTURE**

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A tinted graph is a graph whose arcs are colored with certain colors. A colored graph is a graph whose vertices are colored with certain colors. If \( M \) is the set of tinted (or colored tinted) graphs of order \( k \) and \( G \) is a tinted (or colored tinted) graph, then we shall say that \( G \) is \( M \)-regular (or \( M \)-regularly colored) if all its subgraphs of order \( k \) belong to \( M \). We shall show how, for any formula \( \varphi \) of the first-order predicate calculus, to construct a finite set \( B_\varphi \) of tinted graphs of order 3 and a finite set \( C_\varphi \) of colored tinted graphs of order 2 such that \( \varphi \) if and only if there exists a \( B_\varphi \)-regular tinted graph not admitting a \( C_\varphi \)-regular coloring. Hadwiger's conjecture (HC) is as follows: If no subgraph of a graph without loops \( G \) is contractible to a complete graph of order \( n \), then the vertices of \( G \) can be colored in \( n - 1 \) colors such that neighboring vertices are colored with different colors. We construct a formula \( X \) of the first-order predicate calculus such that HC is equivalent with \( \forall \varphi \rightarrow \neg X \). Thus, HC reduces to HC; if all subgraphs of order 3 of the tinted graph \( G \) belong to \( B_X \), then \( G \) is \( C_X \)-regularly colorable. Here \( B_X \) and \( C_X \) are specific finite sets of tinted graphs of order 3 and colored tinted graphs of order 2, respectively.

**Introduction**

Oriented graphs without loops, multiple and intersecting arcs, all of whose arcs are arbitrarily colored with certain colors, will be called tinted graphs. Graphs, including tinted ones, all of whose vertices are colored arbitrarily with certain colors, will be called colored.

A subgraph of a graph \( G \) will mean a graph formed by a certain subset of the vertices of \( G \) and all arcs both ends of which lie in this subset. If \( G \) is a tinted or colored graph, then its subgraphs are also assumed to be tinted or colored, i.e., the tints of arcs and colors of vertices are preserved.

The order of a graph is the number of its vertices. If \( B \) is a certain set of graphs (possibly tinted or colored), then the graph \( G \) will be called \( B \)-regular if all its connected subgraphs belong to \( B \), and \( B^k \)-regular translated from Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR, Vol. 88, pp. 209-216, 1979. Original article submitted April 25, 1978.
(or $B^k$-regularly colored) if all its connected subgraphs of order $k$ belong to $B$.

We shall show how, for an arbitrary formula $P$ of the first-order predicate calculus, to construct a finite set $B_P$ of tinted graphs of order 3 and a finite set $C_P$ of colored tinted graphs of order 2 (i.e., arcs) such that the formula $P$ is derivable if and only if there exists a $B^3_P$-regular tinted graph, not admitting a $C^2_P$-regular coloring.

Hadwiger's Conjecture (HC). If a nonoriented graph without loops has no subgraphs, contractible to a complete graph of order $n$, then it can be colored in $n - 1$ colors so that neighboring vertices are colored with different colors.

Let $L_n$ be the set of all nonoriented graphs without loops, noncontractible to a complete graph of order $n$, and $E_n$ be the set of all colored graphs, each of which is a nonoriented edge with ends, colored with two different colors from $n$ fixed colors. Let $G$ be a variable for arbitrary finite graphs. Then HC can be formulated in this way:

$$\forall G \ (G \text{ is } L_n\text{-regular} \implies G \text{ is } E^2_n\text{-regularly colorable}).$$

If $X_1$ and $X_2$ are statements of the first-order predicate calculus such that HC $\iff (X_1 \text{ is satisfied})$ or HC $\iff (\neg X_2)$, then, according to what was said above, HC is equivalent with the assertion

$$\forall G \ (G \text{ is } B^3_{1X_1}\text{-regular} \implies G \text{ is } C^2_{1X_1}\text{-regularly colorable})$$

or, respectively,

$$\exists G \ (G \text{ is } B^3_{2X_2}\text{-regular} \land \text{G is not } C^2_{2X_2}\text{-regularly colorable}).$$

We construct a formula $X_1$, having the property indicated. Thus, by replacing $L_n$ by $B^3_{1X_1}$ and $E_n$ by $C_{1X_1}$, we succeed in removing the outmost quantifier with respect to $n$ in Hadwiger's conjecture. Moreover, $B^3_{1X_1}$-regularity, in contrast with $L_n$-regularity, is a "local" property of a graph.

1. Representation of Proofs by Tinted Graphs

Suppose given a formula $P$. We reduce $\forall P$ to Skolem normal form as far as possible, eliminating quantifiers and separating variables in the conjunctive terms of the formula obtained. We get the formula $\forall P \mathcal{D}_i$, where $\mathcal{D}_i$ contains the variables $x_{ij} (j = 1 \ldots i)$. (We note that the disjointness of $\mathcal{D}_i$ is not assumed.) The $k$-th standard variant of $\mathcal{D}_i$ is obtained from $\mathcal{D}_i$ by adding to all variables the upper index $k$ and is denoted by $\mathcal{D}^k_i$. By Herbrand's theorem $\forall P$ if and only if there exists a finite set of formulas $\mathcal{D}^k_i$ and a substitution $\alpha$ such that the formula $\forall P \mathcal{D}^k_i \alpha$ is identically false in the propositional calculus [4].

Definition. If $T$ is some set of terms, and $\approx$ and $<$ are binary relations on it, then we shall say that the substitution $\alpha$ realizes these relations if for any $t_1$, $t_2$ from $T$

$$t_1 \approx t_2 \iff (t_1 \alpha \text{ is graphically equal to } t_2 \alpha),$$

$$t_1 < t_2 \iff (t_1 \alpha \text{ is a strict subterm of } t_2 \alpha).$$

Let $T^k_i$ be the set of terms of the formula $\mathcal{D}^k_i$. By the tint of an arc will be meant a quadruple $(i, j, \approx, <)$, where $\approx$ and $<$ are binary relations on $T^k_i \cup T^k_j \approx$ realized by some substitution. The set of tints of arcs is finite and is defined by the formulas $\mathcal{D}^k_i$. The set $\{\mathcal{D}^k_i\}$ and substitution $\alpha$ we make correspond to a tinted graph in the following way:

a) to each $\mathcal{D}^k_i$ corresponds a vertex $V^k_i$,

b) if for some $\mathcal{D}^k_i$ and $\mathcal{D}^k_j$ realizes on $T^k_i \cup T^k_j$ the relations $\approx$ and $<$, at least one of which is nonempty, then we draw from $V^k_i$ to $V^k_j$ an arc and we let it correspond to the tint $(i, j, \approx, <)$, where the relations $\approx$ and $<$ on $T^k_i \cup T^k_j$ are obtained from the relations $\approx$ and $<$ on $T^k_i \cup T^k_j$ by replacing $k$ by 1 and $l$ by 2.

The opposite arc from $V^k_j$ to $V^k_i$ will not be drawn.

In the tinted graph obtained there can be only those connected subgraphs of order 3 for which the tints of arcs are compatible, i.e., firstly, to the vertices one can attach indices $i_1$, $i_2$, $i_3$ so that any arc going from a