The invariance principle in mathematical statistics, excellently presented in Lehmann's book [5], is well known. In the present paper, we attempt to apply an analogous principle in problems of optimal stopping of Markov processes.

Let $(E, \mathcal{E})$ be a Hausdorff locally compact space with countable basis, together with its Borel $\sigma$-algebra, and let some group $\Gamma$ act on $E$. Let us assume that the factor space $E/\Gamma$ is again a Hausdorff locally compact space with countable basis, and that for any Borel function $f : E \times \mathbb{R} \to \mathbb{R}$ of the form $f = f \circ \varphi$, where $\varphi$ is the canonical mapping of $E$ onto $E = E/\Gamma$ and $\tilde{f} : E \to \mathbb{R}$, $\tilde{f}$ is a Borel function. We will denote the fact that $f$ and $\tilde{f}$ are Borel functions by $f \in \mathcal{F}$ and $\tilde{f} \in \tilde{\mathcal{F}}$ ($\tilde{\mathcal{F}}$ is the Borel $\sigma$-algebra of the space $\tilde{E}$).

Let $X = (\Omega, \mathcal{F}, \mathcal{F}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}_s, \tilde{\mathcal{F}}_s, \tilde{\mathcal{F}}_s, \tilde{\mathcal{F}}_s, \tilde{\mathcal{F}}_s, \tilde{\mathcal{F}}_s)$ be a standard Markov process on the space $(E, \mathcal{E})$, where $\mathcal{F}_s = \sigma(x_s, s \geq 0)$. If $f \in \mathcal{F}$, then $f \circ \varphi \in \mathcal{F}$, and $\mathcal{F}_{s\mu}$ is the completion of the $\sigma$-algebra $\mathcal{F}_{s\mu}$ with respect to the measure $P_{s\mu}$. We will denote analogously all further processes considered. Let the transition function of the process satisfy the following condition:

$$P(t, x, B) = P(t, y - x, y - x B), \quad t \geq 0, \quad x \in E, \quad B \in \mathcal{E}, \quad \gamma \in \Gamma. \quad (1)$$

Then by Theorem 10.13 [2], we can construct the factor process

$$\tilde{X} = (\Omega, \tilde{x}_n, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}_s, \tilde{\tilde{\mathcal{F}}}_s),$$

on the space $(\tilde{E}, \tilde{\mathcal{E}})$, where $\tilde{x}_t = \varphi \circ x_t$, and

$$\tilde{P}_s(A) = P_s(A), \quad \tilde{x} = \varphi(x), \quad A \in \tilde{\mathcal{F}}_s. \quad (2)$$

For the standard process

$$\tilde{x} = (\Omega, \tilde{x}_n, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}, \tilde{\mathcal{F}}_s)$$

it remains to verify that

$$\tilde{\mathcal{F}}_s = \bigcap_{s \geq 0} \tilde{\mathcal{F}}_s.$$

But for this, by virtue of Proposition 1.8.12 [1], it is sufficient that the process $\tilde{X}$ be Markov with respect to

$$\{\tilde{\mathcal{F}}_s\}.$$
The latter follows from the standardness of the process $X$ and the inclusion chain

$$\hat{\mathcal{F}}_t^\circ \subset \mathcal{F}_t^\circ \subset \mathcal{F}_t = \mathcal{F}_t.$$  

**Lemma.**

$$\hat{\mathcal{F}}_t \subset \mathcal{F}_t \text{ and } \hat{\mathcal{F}} \subset \mathcal{F}.$$  

**Proof.** Let $A \in \hat{\mathcal{F}}$. We must prove that for any measure $\mu \in M(\mathcal{E})$ there exist $A_1, A_2 \in \mathcal{F}^\circ$ such that $A_1 \subset A \subset A_2$ and

$$P_\mu(A_2 \setminus A_1) = 0.$$  

Let $\nu$ be the restriction of $\mu$ to the $\sigma$ algebra $\gamma^{-1}(\mathcal{E})$. Then $\nu \in M(\hat{\mathcal{E}})$, and for it, there exist $A_1, A_2 \in \mathcal{F}^\circ$, such that $A_1 \subset A \subset A_2$ and

$$\mathcal{P}_\nu(A_2 \setminus A_1) = 0.$$  

which proves the lemma.

For any function $g \in \mathcal{A}$, we define

$$s(x) = \sup_{\tau \in \mathfrak{M}(\gamma)} E_x g(\tau_x), \quad x \in E,$$

where $\mathfrak{M}(\gamma)$ is the set of stopping times with respect to the $\sigma$ algebra in the parentheses.

**Theorem 1.** Let the function $g \in \mathcal{A}$ satisfy the following conditions:

1) $g = \hat{g} \circ \varphi$, where $\hat{g} : \hat{E} \to \mathbb{R}$,

2) $g$ is lower semicontinuous in the natural topology of the process $X$,

3) $E_x [\sup_{t \geq 0} g^*(x_t)] < \infty$, $x \in E$.

Then

$$s(x) = \hat{s} (\varphi(x)) = \sup_{\tau \in \mathcal{M}(\gamma)} E_{\tau_x} \hat{g}(\tau_x).$$

Proof. It is known ([3, 7]) that $s$ is the least excessive majorant (l.e.m.) of the function $g$ for the process $X$. From the construction of the l.e.m. ([3, 7]), by virtue of Theorem 10.13 [2],

$$Q_\mu g = \max(g, T_\mu g) = \max(g \circ \varphi, \hat{T}_\mu g \circ \varphi) = \left( \max(g, \hat{T}_\mu g) \right) \circ \varphi = \hat{Q}_\mu g \circ \varphi,$$

where $T_\mu$ and $\hat{T}_\mu$ are the semigroups of operators of the processes $X$ and $\hat{X}$. Hence,

$$v_\mu = \lim_{N \uparrow \infty} (Q_\mu^N g) = \lim_{N \uparrow \infty} (Q_\mu^N \hat{g} \circ \varphi) = \left( \lim_{N \uparrow \infty} \hat{Q}_\mu^N \hat{g} \right) \circ \varphi = \hat{\varphi} \circ \varphi.$$

By virtue of the assumptions made with regard to the action of the group $\Gamma$, the conditions of the theorem, and Eqs. (8)-(10), it follows that $\hat{v}$ is the l.e.m. of the function $\hat{g}$ for the process $\hat{X}$, and $\hat{v} = \hat{s}$. 

$g^- = \max(0, -g)$. 