SIMULTANEOUS BUNDLE ADJUSTMENT OF THE QUADRUPLET

ZBYNĚK MARŠÍK

Research Institute of Geodesy, Prague*

Summary: The quadruplet of aerial photographs with 60% end and side overlaps is the minimum block where the orientation of photographs is to be adjusted. Simultaneous adjustment of the relative orientation of the four photographs is now suggested and the necessary equations are derived.

1. THE RELATIVE ORIENTATION OF PHOTOGRAPHS IN THE QUADRUPLET

Four aerial photographs with 60% end and side overlaps are the smallest possible block of photographs, which allow us to call it a quadruplet. It is right to speak of a block, because the geometrical characteristics of this quadruplet are sufficient for the adjustment of the relative orientation of the four photographs. A new procedure of block adjustment was suggested in [2], where the quadruplet was applied as a subblock. It was explained in the paper mentioned, how to form the block of the quadruplets, and also how to adjust the relative orientation of the quadruplet. The traditional way of the relative orientation of two photographs was applied; deviations $\kappa, \varphi, \omega$ of the closure enabled one to find the corrections for the orientation of each photograph. However, the simultaneous adjustment of the relative orientation of all four photographs is possible. In the present paper an idea of the solution is given, the geometric relations are established, and the necessary equations are derived.

2. ADJUSTED RELATIVE ORIENTATION OF BUNDLES

Consider the quadruplet of aerial photographs shown in Fig. 1. Photograph 11 is regarded to be stable, all other photographs can be oriented relatively to it. One of the orientation elements, e.g. $bx'$, is optional; 17 orientation elements remain to be found by computation. An iterative procedure for the solution is suggested, where the bundle of rays is the basic unit.

The bundles can be reconstructed, or in other words, the direction cosines of each ray of light can be computed, when the image coordinates $x', y'$ and the focal length $f$ are known. For the sake of later derivations it is useful to compute the fractions

\begin{equation}
A_{i,n} = \frac{\cos \alpha_{i,n}}{\cos \gamma_{i,n}} = \frac{x_{i,n}'}{f}, \quad B_{i,n} = \frac{\cos \beta_{i,n}}{\cos \gamma_{i,n}} = \frac{y_{i,n}'}{f},
\end{equation}

where the subscript $i$ identifies the photograph and the subscript $n$ the point $P$.

The principles of adjustment of model coordinates, given in [1], are applied to the procedures which follow. The vector $d$ in Fig. 1 is the distance between the adjusted

*) Address: Nádražní 31, 150 79 Praha 5.

Studia geoph. et geod. 19 [1975] 115
position of the point \( P_n \) and the point of intersection of one ray of light with a certain plane \( XY \). The adjusted coordinates \( X_{P_n}, Y_{P_n}, Z_{P_n} \) are computed with regard to all rays of light corresponding to the model point \( P_n \). For each point \( P_n \) a plane \( XY \) is sought, which has a height \( Z \) such that the points of intersection of the corresponding rays with that plane will have the minimum dispersion. The adjusted position of \( P_n \) will be determined under the condition \( \Sigma(dd) = \min \). In [1] it was shown that the vector

\[
\mathbf{d} = \mathbf{i}(X_{P_n} - X_i) + \mathbf{j}(Y_{P_n} - Y_i) - \mathbf{i}(Z_{P_n} - Z_i) A_i^{t,n} - \mathbf{j}(Z_{P_n} - Z_i) B_i^{t,n},
\]

and by using the known procedure from the least-squares adjustment, the normal equations were derived to read

\[
\begin{align*}
mx_{P_n} - Z_{P_n}S_{ac} - S_x &= 0, \\
+ my_{P_n} - Z_{P_n}S_{bc} - S_y &= 0, \\
-X_{P_n}S_{ac} - Y_{P_n}S_{bc} + Z_{P_n}S_{abc} - S_z &= 0,
\end{align*}
\]

where \( S \) are symbols representing the sums

\[
\begin{align*}
S_{ac} &= \Sigma A_i^{t,n}, \\
S_x &= \Sigma(X_i - Z_iA_i^{t,n}), \\
S_{bc} &= \Sigma B_i^{t,n}, \\
S_y &= \Sigma(Y_i - Z_iB_i^{t,n}), \\
S_{abc} &= \Sigma(A_i^{t,n} + B_i^{t,n}), \\
S_z &= \Sigma(X_iA_i^{t,n} + Y_iB_i^{t,n} - Z_i(A_i^{t,n} + B_i^{t,n}))
\end{align*}
\]

and \( m \) is the number of rays corresponding to one point \( P_n \). When the values \( A_i^{t,n}, B_i^{t,n} \) computed according to (1) are substituted into (3), the approximate coordinates \( X_{P_n}, Y_{P_n}, Z_{P_n} \) of each point \( P_n \) will be found. The next step of the iterative procedure is to find an orientation of each of the three variable bundles such that the corresponding rays will have the smallest possible deviations with regard to all points \( P_n \). When the new orientations of the three bundles are determined, the second iterative cycle can start by computing the points \( P_n \), and the procedure continues as before. It is known from experience that three to four iterative computations are necessary to determine the relative orientation of two photographs. For the simultaneous orientation of four photographs more iterations can be expected; the number of iterations has to be found experimentally.

The optimum orientation of the variable bundles, regarding all points \( P_n \), can be carried out in various ways. It seems to be logical to use the same principle as for the computation of points \( P_n \). The derivation starts with equation (2) again. Fig. 1 helps the reader to understand the idea. If the coordinates of the projection center of the stable photograph are denoted by \( X_0, Y_0, Z_0 \) and the elements of the photogrammetric bases by \( bx^i, by^i, bz^i \), the other three projection centers will have the coordinates \( X_i = X_0 + bx^i, Y_i = Y_0 + by^i, Z_i = Z_0 + bz^i \), which can be substitut-