Combined heat and mass transfer in natural convection between vertical parallel plates

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Abstract. This study purposes to examine the effects of latent heat transfer associated with the liquid films vaporization on the heat transfer in the natural convection flows driven by the simultaneous presence of combined buoyancy forces of thermal and mass diffusion. Results are especially presented for an air-water system under various conditions. The influence of channel length and system temperatures on the momentum, heat and mass transfer in the flow are investigated in great detail. The important role of transport of latent heat of vaporization under the situations of buoyancy-aiding and opposing flows is clearly demonstrated.

Nomenclature

- $A$: $[(C_{p1} - C_{p2})/C_p](w - w_0)$
- $b$: half channel width
- $C_p$: specific heat
- $D$: mass diffusivity
- $D_h$: hydraulic diameter ($= 4b$)
- $g$: gravitational acceleration
- $Gr_M$: reference Grashof number (mass transfer)
- $Gr_M^r$: reference Grashof number (mass transfer)
- $Gr_T$: Grashof number (heat transfer)
- $Gr_T^r$: reference Grashof number (heat transfer)
- $h_{fg}$: latent heat of vaporization
- $h_M$: local mass transfer coefficient
- $k$: thermal conductivity
- $l$: channel length
- $M$: molecular weight
- $Nu_1$: local Nusselt number (latent heat)
- $Nu_w$: local Nusselt number (sensible heat)
- $Nu_x$: overall local Nusselt number
- $p$: pressure of the moist air in the channel
- $P$: dimensionless motion pressure (pressure defect)
- $p_m$: motion pressure (pressure defect), $p - p_0$
- $p_0$: ambient pressure
- $Pr$: Prandtl number, $v/ð$
- $p_w$: partial pressure of water vapor at interface
- $Q$: total heat transfer rate
- $Q_{w0}$: total heat transfer rate without liquid water film
- $q^*$: interfacial energy flux flowing into air stream
- $Re$: Reynolds number at the inlet, $u_0D_h/v$
- $S$: parameter, Eq. (16)
- $Sc$: Schmidt number, $v/ð$
- $Sh_a$: local Sherwood number
- $T$: temperature
- $u$: axial velocity
- $u_0$, $U_0$: average inlet velocity
- $U$: dimensionless axial velocity
- $v$: transverse velocity
- $V$: dimensionless transverse velocity
- $w_i$: mass fraction of water vapor
- $W$: dimensionless mass fraction of water vapor
- $w_s$: saturated mass fraction of water vapor at $T_w$ and $p_0$
- $x$: axial coordinate
- $X$: dimensionless axial coordinate
- $y$: transverse coordinate
- $Y$: dimensionless transverse coordinate
- $z$: thermal diffusivity
- $ð$: dimensionless temperature, $(T - T_0)/(T_w - T_0)$
- $ð'$: dimensionless temperature, $(T - T_0)/(T_0 - T_w)$
- $v$: kinematic viscosity
- $q$: density
- $φ$: relative humidity of air at ambient condition

Subscripts

- 1: of water vapor
- 2: of air
- $w$: condition at interface
- 0: at inlet condition
- $r$: at reference condition

1 Introduction

Energy transports in the natural convection flows driven by the combined buoyancy forces of heat and mass transfer resulting from simultaneous presence of differences in temperature and variations in concentration are important in...
natural environments and engineering applications. Noticeable examples include double-diffusion convection in ocean flows, the simultaneous diffusion of metabolic heat and perspiration in controlling our body temperature especially in hot summer days, the cooling of a high temperature surface by coating it with phase-change material and the process of the evaporative cooling for waste heat disposal.

The natural convection heat transfer in vertical open channel flows induced by the buoyancy force of thermal diffusion only has been studied in great detail [1—4].

The effects of the mass diffusion on laminar natural convection flows have been widely studied for flows over plates with different inclinations [5, 6] and for flows over vertical cylinders [7]. By employing similarity transformation, Hason and Mujumdar [8] investigated the flow along a vertical circular cone. They found that the heat transfer and friction coefficients increase when the concentration buoyancy force aids the thermal diffusion force. The reverse is true as the buoyancy forces are opposed to each other. Recently, natural convection in a vertical channel with opposing buoyancy forces was investigated by Lee et al. [9]. In their studies, the constant flow rate of CO2 was injected from the channel wall to the flow. The effects of injection rate and wall temperature on the flow characteristics were discussed in detail.

The reviews mentioned about for the studies of heat and mass transfer in natural convection flows, however, have restricted their considerations to the external flow systems (except that of Lee et al. [9]). Recognizing the relatively little research on the internal flow systems, the present authors [10] performed an analysis of the natural convection in vertical open tube resulting from the combined buoyancy forces of thermal and mass diffusion, particularly for air-water vapor mixture.

Despite the natural convection due to the combined buoyancy forces of heat and mass diffusion between vertical parallel plates is relatively important in the applications encountered in the engineering systems, it has not received enough attention. Consideration is given to, in the present study, the effects of latent heat transfer, in connection with the vaporization of the thin liquid film on the channel surface, in natural convection flows driven by the simultaneous presence of combined buoyancy effects of the thermal and mass diffusion.

2 Analysis

The system to be examined, as schematically shown in Fig. 1, is the vertical parallel plates with channel length l and half channel with b. The channel surface is wetted by the thin liquid water film. The loss of evaporated liquid in the films is compensated by the injection of additional liquid through the porous channel plate. Under the good control of the liquid injection and appropriate choice of the porosity of porous channel plate, the film on the channel’s inside surface can be maintained so thin that channel surface is just wetted.

In reality, the liquid film is finite in thickness. The film could move upwards or downwards, and the shape of the liquid-gas interface could be quite complex. As a result, the momentum, heat and mass transfer in the film should also be examined with the interfacial phenomena appropriately treated. This would, however, greatly complicate the study and make the theoretical work formidable. The influences of the finite liquid film on the heat transfer may not be properly assessed without the help from the experimental measurement which will be conducted shortly. As a preliminary attempt, the treatment of complex flow motion in the liquid film and at the interface is circumvented by assuming the film being extremely thin so that it can be regarded as a boundary layer condition for heat and mass transfer - the film is stationary and at the same uniform temperature with the channel wall Tw. The moist air in the ambient is drawn into the channel by the combined buoyancy forces resulting from the nonuniformities in temperature and in concentration of water vapor in the flow between the channel plates and the ambient. Since the molecular weight of water vapor is smaller than that of air, the buoyancy force due to mass transfer acts upwards. Consequently, the flow induced by the buoyancy force of thermal diffusion is aided by the buoyancy force of mass diffusion if Tw > To. While the flow is opposed if Tw < To.

By introducing the Boussinesq’s approximation (the concentration of water vapor in the mixture being very low and the temperature differences in the system being small), the steady laminar natural convection flow of moist air in a vertical channel resulting from the combined buoyancy effects of thermal and mass diffusions can be described by the basic equations, in dimensionless form, as:

continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

(1)

axial-momentum equation

$$\frac{U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{dP}{dX} + \frac{\partial^2 U}{\partial Y^2} \pm \frac{Gr_T \cdot \theta + Gr_M \cdot W}{Gr_T + Gr_M}.$$  
(2)