Renormalizable Models
with Simple Symmetry Breaking

I. Symmetry Breaking by a Source Term

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Abstract. If to a Lagrangian density with invariance under a continuous group of linear
transformations of the fields a term linear or bilinear in the fields is added, the symmetry is
in general reduced and the currents associated with the original symmetry are only partially
conserved. If the theory without the added term is renormalizable, the theory with that
term also is, and the needed renormalization conditions are the essential content of the
appropriate Ward-Takahashi-Kazes-Rivers identities. The case of symmetry breaking
by a term linear in Bose fields (source term) is here analysed completely, in particular with
respect to the nonsymmetric limit of vanishing source term, a particular Goldstone mode,
and with respect to properties of the ground state energy density as a function of the strength
of the source term. Induced and spontaneous breaking of a discrete symmetry are also
treated.

Introduction

B. W. Lee [1] has discussed the sigma model [2, 3] from the point of
view of renormalized perturbation theory, in order to have available a
model that satisfies PCAC\(^1\) and allows to calculate in a formal but con-
sistent way the amplitudes for processes involving nonsoft pions.

We shall show here\(^2\) that for such models the renormalized perturba-
tion expansions can be very simply obtained if the relations, stemming
from PCAC, between vertex functions of different numbers of arguments
are exploited. These relations yield all the renormalization conditions
required in Bogoliubov-Parasiuk-Hepp (BPH)\(^3\) renormalization theory
in terms of only that many parameters as the unrenormalized Lagrangian
has. This technique also covers the Goldstone mode\(^4\) obtained in the limit
of vanishing source but, since it deals with renormalized quantities only,

\(^1\) Ref. [4] discusses the sigma model and related models from the point of view of
applications to pion physics.
\(^2\) A short account was given in Ref. [5].
\(^3\) See Ref. [6] and references given there.
\(^4\) Ref. [7] gives a comprehensive presentation of the relevant material.
allows no direct conclusion concerning e.g. whether, switching off the source term while leaving the rest of the Lagrangian unchanged, the ground state for the latter would be the usual symmetric or the non-symmetric Goldstone one, although this question is meaningful.

The Goldstone situation can in an intuitively appealing way be illuminated in terms of the behaviour of the ground state energy density as a function of the source strength. The discussion hereto, familiar in the classical case, carries over with few changes to quantum field theory, whereby, however, a formal similarity to the theory of condensation of Yang and Lee [8] is noted.

In Section I, the well-known one-particle structure of Green's functions is presented concisely. In Section II, for Green's functions involving current operators that have simple commutators with the fields, Ward-Takahashi-Kazes- and Rivers-type identities are derived. In Section III, the formulas of the first two sections are written for the special case of a symmetric Lagrangian density with added term linear in Bose fields. These relations are used in Section IV to obtain the BPH renormalization conditions in terms of only the number of parameters that appear in the unrenormalized Lagrangian. The same is done, with some necessary precaution to avoid spurious infrared divergences, in Section V for the associated Goldstone mode, i.e. the limit theory with vanishing source and spontaneously broken symmetry, which may also be described directly in terms of a manifestly nonsymmetric Lagrangian. In Section VI, the relation between the theories with and without symmetry breaking source term is discussed and a comparison is made with the breaking of a discrete symmetry. The appendix contains the discussion of the properties of the ground state energy as a function of the source strength, which are obtained using results of Euclidean quantum field theory.

The calculation of the Green's functions involving a current operator will, because of the technique needed hereby, be included in the sequel paper, which deals with symmetry breaking by a term bilinear in fields.

I. One-Particle Structure of Green's Functions

We wish to consider the Poincaré-invariant theory of a multi-component local hermitean field $A(x)$ described by the Lagrangian density

$$L = L(A, \partial A).$$

(I.1)

To this end we consider, following Schwinger [9], the related theory described by the Lagrangian density

$$L' = L(A', \partial A') + J A'$$

(I.2)