Currents, Stress Tensor and Generalized Unitarity in Conformal Invariant Quantum Field Theory

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Abstract. Matrix elements of internal symmetry currents and energy momentum density tensor are constructed in Migdal Polyakov conformal invariant bootstrap field theory. Their 3-point functions satisfy Bethe Salpeter equations which determine any free coefficients that may still occur in the conformal invariant Ansatz. Ward identities are verified for all n-point functions. They imply correct equal time current commutation relations. A proof of generalized unitarity is also given. Various equivalent forms of the propagator bootstrap are discussed. Our algebraic techniques also yield an eigenvalue equation for first order correction to the exactly conformal invariant theory, assuming the latter is Gell-Mann Low large momentum asymptote of a renormalizable finite mass theory.

Introduction

The Migdal-Polyakov bootstrap approach [1−5] to construct a conformal invariant quantum field theory offers an interesting alternative to the up to now only available canonical perturbation theory. This new approach has been shown [5] to be free from UV and infrared divergences. Moreover, it may be hoped to reproduce correctly the behavior of realistic strong interaction quantum field theory (QFT) in a selected class of high energy limits, since it appears [6] that the Gell-Mann Low limit [7, 8] of perturbation theoretically renormalizable theories indeed is conformal invariant.\footnote{The crucial point is the softness for the trace of the stress tensor in the finite mass theory, which was proven recently by Schroer [6]. Use of conformal symmetry was advocated long ago by Wess and Kastrup [9]. That the divergence of the dilatation current should be a soft operator, i.e. emphasize low frequencies, was first conjectured in Ref. [10]. The discussion was extended to conformal symmetry in Ref. [11]. The hypothesis became powerful after Wilson combined it with the notion of operator product expansion [12].}

For the physical interpretation of the theory one should show how to extract observables. Since one is dealing with an infraparticle theory...
[13] an $S$-matrix does not exist [12]. Therefore, and also as a further check of the consistency of the theory, one looks for other observables characteristic for local QFT: local charge and current, and local energy momentum, i.e. a stress tensor. These should satisfy the appropriate Bethe-Salpeter equations to guarantee that they are local operators, and Ward-Takahashi identities which identify them.

Furthermore, if the theory is a local QFT it must also satisfy the axiomatic positivity requirements, for which generalized unitarity [14] is sufficient. One knows that a massive theory does satisfy generalized unitarity if its Green functions have the many-particle structure [15] familiar from canonical perturbation theory. For the many-point Green functions, these structure properties are manifest in skeleton expansions. For the vertex and the propagator, they are assured by their integral equations (here called bootstraps). Generalized unitarity will then also hold for the Gell-Mann Low limit theory if the massive theory considered is a renormalizable Lagrangian one.

However, it seems nevertheless desirable to attempt a direct verification of generalized unitarity for the conformal invariant Migdal-Polyakov theory since one does not want to start with the assumption that it is Gell-Mann Low limit of a massive theory.

These are the problems to be studied in the present paper. In going along, we shall also have occasion to study integral equations for the 3-point functions and, especially, the 2-point functions [16]. They provide convenient alternative forms of the propagator bootstrap, more tractable than unitarity type relations because analytic continuation to euclidean space [17] can be performed. This avoids the difficulties inherent in the use of conformal invariance in Minkowski space [18].

Conformal invariant QFT is a noncanonical theory. It relies on Wilson's idea [12] of dynamical dimensionality of fields, which in turn signals the breakdown of canonical equal-time commutation relations. Therefore, one cannot use the canonical formalism to infer existence of conserved local currents associated with symmetries. An explicit demonstration is needed.

We will explain in detail how to construct Green functions involving internal symmetry currents (Section 2) or the stress energy tensor (Section 3). It will be proven in Section 4 that all the Green functions so constructed satisfy the appropriate Ward Takahashi identities, as a consequence of the bootstrap equations stated in Section 1. We remark that Ward identities are not true skeleton graph by skeleton graph, as

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2 A demonstration of gauge invariance in renormalized perturbation theory which does not use the canonical formalism (i.e. a regularized Lagrangian) has been given by Brandt [19] for Q.E.D. Some of our algebra has been inspired by his.