Some Consequences of Infinite Mass Renormalization

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Abstract. Using a finite form of local field equations some consequences of infinite mass renormalization are studied in a rigorous manner. The method is applied to various models. For pseudoscalar meson-nucleon interaction sufficient conditions are given for the equivalence to a direct Fermi coupling. Confirming a recent result by KROLL, LEE, and ZUMINO it is shown that a vector meson field should be proportional to the corresponding current if the bare meson mass is infinite. In the conventional treatment of neutral vector meson theories this causes certain difficulties which are analyzed in detail. In case of two vector fields coupled to the same current it is found that the fields must be proportional provided the bare masses are both infinite. In the Appendix finite local field equations are discussed for the coupling of a neutral vector meson field to the current of a spin 1/2 field.

1. Introduction

To illustrate the purpose of this paper we consider the model of $A^4$-coupling. The model is described by a scalar, Hermitian field $A(x)$ which is an operator valued distribution

$$A_f = \int f(x) A(x) \, dx$$

defined for a suitable class of test functions on a dense linear subset $D$ of Hilbert space. $A(x)$ is supposed to satisfy the general principles in Wightman's formulation of relativistic, local quantum field theory [1-3]. In addition we assume that $A(x)$ satisfies the field equation

$$-(\Box + m^2) A(x) = \lim_{\xi \to 0} j(x, \xi)$$

$$j(x, \xi) = \lambda Z_1(\xi) Z_2^{-1}(\xi) : A(x + \xi) A(x) A(x - \xi) : - \delta m^2(\xi) A(x)$$

which has been verified in renormalized perturbation theory [8]. The product is defined by the usual vacuum subtractions

$$:A(x_1) A(x_2) A(x_3):$$

$$= A(x_1) A(x_2) A(x_3) - \langle A(x_1) A(x_2) \rangle_0 A(x_3) - \text{cycl. perm.}$$

for

$$(x_i - x_j)^2 < 0 \; \; (i \neq j).$$

1. Concerning the choice of the class of test functions see JAFFE's discussion [3].
2. Field equations of this type were first proposed by VALATIN [4] for quantum electrodynamics. The theory has been further developed and extended to other interactions in ref. [5-9].
3. Throughout this paper $\lim_{\xi \to 0}$ will denote the spacelike limit with $\xi^2 < 0$ and $\xi^0 / -\xi^2$ bounded.
\( m \) and \( \lambda \) denote the (finite) mass and coupling constants. Formally the values of the functions \( Z_1, Z_2 \) and \( \delta m^2 \) at \( \xi = 0 \) represent the renormalization constants of vertex function, wave function and mass respectively.\(^4\)

The limit \( \xi \to 0 \) should be understood in the weak sense that (1.1) holds for matrix elements

\[
(\Phi_1, \ldots, \Phi_2) \quad \text{with} \quad \Phi_1 \in D_1, \ \Phi_2 \in D_2. \tag{1.3}
\]

\( D_1 \) and \( D_2 \) are dense linear subsets of \( D \) with the property that

\[
(\Phi_1, A(x) \Phi_2), \ (\Phi_1, A(x+\xi) A(x) A(x-\xi) \Phi_2)
\]

are ordinary functions of \( x \) and \( \xi \) for

\[
\Phi_1 \in D_1, \ \Phi_2 \in D_2 \quad \text{and} \quad \xi^2 < 0.
\]

We will derive a simple consequence of the assumption that the mass renormalization is infinite, i.e.

\[
|\lim_{\xi \to 0} \delta m^2(\xi)| = \infty \tag{1.4}
\]

Equ. (1.1) may be written in the equivalent form

\[
- (\Box + m^2) A(x) = j(x, \xi) + o(x, \xi) \tag{1.5}
\]

where

\[
\lim_{\xi \to 0} o(x, \xi) = 0.
\]

Dividing both sides of (1.5) by \( \delta m^2(\xi) \) and taking the limit \( \xi \to 0 \) we obtain with (1.4) the relation

\[
A(x) = \lambda \lim_{\xi \to 0} Z_1(\xi): A(x+\xi) A(x) A(x-\xi): \frac{Z_2(\xi)}{\delta m^2(\xi)} \tag{1.6}
\]

for matrix elements of the form (1.3). As should be clear from the derivation equ. (1.6) is an identity which follows from the field equation provided the mass renormalization is infinite.\(^5\) Moreover, (1.6) must be valid in perturbation theory.

\(^4\) In terms of the functions \( \alpha, \beta, \gamma \) of ref. [8] the functions \( Z_1, Z_2 \) and \( \delta m^2 \) are defined by

\[
Z_1(\xi) = \gamma(\xi)^{-1}, \ Z_2(\xi) = 1 + \frac{\beta(\xi)}{\gamma(\xi)}
\]

\[
\delta m^2(\xi) = \lambda \frac{\alpha(\xi)}{\beta(\xi) + \gamma(\xi)}.
\]

\(^5\) It should be noted that (1.6) does not imply the relation

\[
(*) \quad [A(x) A(y)] = \lambda \lim_{\xi \to 0} \left[ \frac{Z_1(\xi): A(x+\xi) A(x) A(x-\xi):}{Z_2(\xi) \delta m^2(\xi)}, A(y) \right], \ x^0 = y^0
\]

which would be inconsistent with the canonical commutation relations.

\((*)\) does not follow from (1.6) because (1.6) is restricted to matrix elements (1.3) for the field equation (1.1) is correct. For instance, in order to derive the vacuum expectation value of \((*)\) one would need (1.1) for matrix elements

\[
(**) \quad (\Omega, \ldots, \phi) \quad \text{and} \quad (\phi, \ldots, \Omega)
\]

\[
\phi = \int d^4y \hat{A}(y) f(y) \hat{\Omega}.
\]

But even if \( Z_2^{-1} \) were finite the left hand side of (1.1) would not be defined for \((**)(*)\) unless \( \int \kappa^4 g(\kappa^4) d\kappa^4 < \infty \) (\( g \) denotes the weight function of the propagator).\(^5\)