Iterated Crossed Box Diagram
in the Complex Angular Momentum Plane
and Bethe-Salpeter Equation*

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Abstract. The analytic properties of the iterated crossed box diagram are considered in the complex angular momentum plane by using the partial wave Bethe-Salpeter equation.

It is shown that the kernel of the Bethe-Salpeter equation is of the Hilbert-Schmidt type in the domain

\[ \mathcal{A}(l, s) = \{l, s; \text{Re } l > -3/2; \text{Im } s \neq 0 \} \]

having a simple pole at \( l = -1 \).

This pole produces in the whole amplitude a singularity found by GRIBOV and POMERANCHUK since the kernel is not of finite rank.

I. Introduction

In the last few years [1] the theory of the complex angular momentum has been a very important and powerful tool for treating the scattering amplitude at high energies.

One way [2] to introduce, in relativistic scattering theory, the concept of the complex angular momentum is by an analytic continuation of the partial wave amplitude into the complex angular \( \ell \) plane. Then the uniqueness [1] of such an analytic continuation requires that the continued amplitude is bounded and analytic in some region of the complex \( \ell \) plane. Generally, that can be satisfied if for the original amplitude before the analytic continuation the validity of a dispersion relation has been assumed.

Thus an analytically continued partial wave amplitude will be analytic in at least a half-plane \( \text{Re } \ell > \ell_0 \), where \( \ell_0 \) is connected with the necessary number of subtractions in a dispersion relation.

We consider the scattering of identical neutral spinless particles with equal mass. The relativistic amplitude written as \( T(s, t) \) may have,
of course, both left- and right-hand cut in the $t$ variable with respect to the channel where $t$ is energy. Then we shall say that our scattering amplitude has a third double-spectral function, following the terminology usual for the Mandelstam representation [3]. In that case the partial wave amplitude $T_{\ell}(s)$, where $s$ is the total center-of-mass energy squared, will be defined [1] by two different analytic functions of $\ell$ which coincide respectively with $T_{\ell}(s)$ for even and odd values of $\ell$.

For the elastic scattering in the $s$-channel when $s$ is in the elastic region one may continue the two particle unitarity condition into the complex $\ell$ plane, thus getting generalized two particle unitarity [4].

The important fact which follows when $T_{\ell}(s)$ satisfies the generalized two particle unitarity is that $T_{\ell}(s)$ is bounded for $s \leq 4m^2$, where $m$ is the particle's mass.

If one goes to the left of the branch point $s = 4m^2$ then from generalized unitarity it also follows that $T_{\ell}(s)$ cannot be unbounded on both Riemann sheets. This means that $T_{\ell}(s)$ cannot have an $s$-independent pole [1] at the point $\ell = \ell_1 < \ell_0$ which one may reach by analytic continuation in the complex $\ell$ plane.

When the third double-spectral function is present then the individual terms of the perturbation series of the partial wave amplitude will have poles at the negative integer values of $\ell$ due to the properties of the $Q_\ell$ function. Here, we have been ignoring possible subtraction in the dispersion relation.

If the whole amplitude satisfies generalized two particle unitarity the perturbation series will then develop singularities, found by GRIBOV and POMERANCHUK [5], at the negative integer values of $\ell$, which will cancel these fixed poles. This simply means that if the such perturbation series is obtained by an iteration process, each term will have poles of increasing order at the above values of $\ell$.

The simplest case of the perturbation series [1] which has the above behaviour may be constructed by iterating the crossed box diagram.

This case we shall consider in the complex $\ell$ plane in some detail in this paper.

When one iterates the crossed box diagram each term will have a pole at $\ell = -1$ and since the whole amplitude, obtained by summing up all iterated terms, satisfies generalized two particle unitarity this pole should disappear.

One may try to see how this cancellation could happen by using the method for finding the high energy behaviour of Feynman graphs [6, 7], and then summing up the most singular terms.

The result will be that the structure of the coefficient $c_N(s)$ of the variable $t$ tending to infinity, for say $N$ times iterated crossed box