The $\beta$-Moments for the Second-Forbidden $\beta$-Decay from $^{137}$Cs, the CVC Theory, and the Residual Interactions

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Abstract. The $\beta$-moments of the $\frac{7}{2}^+ (1.176 \text{ MeV} \beta^-) \frac{5}{2}^+$ transition in the decay of $^{137}$Cs have been extracted from the available experimental data. We have obtained the following values:

$$\langle R_{ij}\rangle/R^2 = 0.035 \pm 0.020, \quad \langle iT_{ij}\rangle/R^2 = 0.067 \pm 0.036, \quad \text{and} \quad -0.085 \leq \langle S_{ij}\rangle/R^2 \leq 0.085.$$  

These results show that the $\beta$-moments are reduced with respect to those evaluated by means of the conventional pairing model. These quenching effects are similar to those previously found in the analyses of first-forbidden $\beta$-decays and are attributed to residual charge-exchange interactions. The results for $A(= -\langle iA_{ij}\rangle/\xi \langle R_{ij}\rangle)$ are discussed in the light of the predictions of the CVC theory.

1. Introduction

Since some years ago it was shown that the phenomenological values of $\gamma$- and $\beta$-moments for spherical nuclei deviate from the calculated values within the framework of the conventional models. Hindrance effects have been found on the nuclear matrix elements (n.m.e.) corresponding to the charge-exchange $E1 \gamma$-transitions in medium and heavy nuclei [1–7]. Similar effects have also been observed on the $\beta$-moments corresponding to the first-forbidden $\beta$-transitions in the same regions of the periodic table [8–15]. These quenching can be attributed to the particle-hole correlation (core polarization) induced by the isospin-dependent residual interactions [1, 4, 9, 16, 17]. All these results have already been discussed in terms of effective coupling constants in previous papers [11–15]. The influence of core polarization on $M2$ and $E3 \gamma$-transitions has been recently investigated for single-proton states in the $N \approx 82$ region [18–20]. In the case of $M2 \gamma$-transitions hindrance effects have been found, and could be explained by the spin-isospin core polarization. On the other hand the study of $E3 \gamma$-transitions has shown an enhancement of the $\gamma$-moments, which could be interpreted as an effect of the octupole core polarization. All the above-mentioned $\gamma$- and $\beta$-moments correspond to transitions between nuclear states with opposite parity ($\Delta \pi = \text{yes}$).

Quite recently, Ejiri et al. [21] investigated the influence of a core polarization corresponding to an operator with $\Delta \pi = \text{no}$. Namely, they have analysed the magnetic moment ($M1$) of the $1h_{11/2}$ single-proton state in $^{141}$Pr. The result indicates that there are also quenching effects in this case. It would be desirable to know more about the possible influence of core polarization on observables described by operators which require $\Delta \pi = \text{no}$. The second-forbidden $\beta$-transitions give an excellent opportunity to search this matter.

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Table 1. Nuclear matrix elements of second-forbidden $\beta$-decays

<table>
<thead>
<tr>
<th>Notation</th>
<th>Nuclear matrix elements</th>
<th>Cartesian</th>
<th>Normalization</th>
<th>Spherical</th>
<th>Multi-polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta x$</td>
<td>$g_{v} \langle R \rangle$</td>
<td>$- \left( \frac{8\pi}{15} \right)^{1/2} (2J_i+1)^{-1/2}$</td>
<td>$\langle J</td>
<td>g_{v} T_{220}^+</td>
<td>\rangle$</td>
</tr>
<tr>
<td>$\eta y$</td>
<td>$g_{v} \langle i T \rangle$</td>
<td>$- \left( \frac{16\pi}{5} \right)^{1/2} (2J_i+1)^{-1/2}$</td>
<td>$\langle J</td>
<td>g_{v} T_{221}^+</td>
<td>\rangle$</td>
</tr>
<tr>
<td>$\eta y$</td>
<td>$g_{v} \langle A \rangle$</td>
<td>$- \left( \frac{16\pi}{3} \right)^{1/2} (2J_i+1)^{-1/2}$</td>
<td>$\langle J</td>
<td>g_{v} T_{221}^+</td>
<td>\rangle$</td>
</tr>
<tr>
<td>$\eta z$</td>
<td>$g_{A} \langle S \rangle$</td>
<td>$- \left( \frac{96\pi}{5} \right)^{1/2} (2J_i+1)^{-1/2}$</td>
<td>$\langle J</td>
<td>g_{A} T_{221}^+</td>
<td>\rangle$</td>
</tr>
</tbody>
</table>

Actually, the second-forbidden $\beta$-transitions are from the theoretical point of view, at least, of the same interest as the first-forbidden ones. When terms induced by the strong interactions and other higher-order corrections are neglected, all the observables for the nonunique second-forbidden $\beta$-transitions can be written in terms of four $\beta$-moments [22], listed in Table 1. None relativistic axial vector $\beta$-moment exists among them. This is an advantage because a reliable way for the theoretical evaluation of such n.m.e. is not available. On the other hand, there exists the relativistic vector $\beta$-moment $\langle i A \rangle$. However in this case, it is possible to find a relation between $\langle i A \rangle$ and the nonrelativistic vector $\beta$-moment $\langle R \rangle$, on the basis of the conserved vector current (CVC) theory [23]. Thus the study of second-forbidden $\beta$-transitions, also allow to test the predictions of the CVC theory.

Although their theoretical importance, the analyses of second-forbidden $\beta$-transitions are not as numerous as those of first-forbidden ones. This is mainly due to the scarcity of such $\beta$-decays along the periodic table. Some years ago Lipnik and Sunier [22] analysed some second-forbidden $\beta$-transitions. Because of the rather simple structure of the nuclear states involved, the $\frac{3}{2}^+(1.176\text{ MeV}\ \beta^-)\frac{3}{2}^+$ transition from $^{137}\text{Cs}$ is one of the most interesting cases. Both nuclear states, i.e., $\frac{3}{2}^+$ in $^{137}\text{Cs}$ and $\frac{3}{2}^+$ in $^{137}\text{Ba}$, are ground states. And they are well described in the framework of the quasiparticle plus $Q-Q$ phonon model (KS) [24, 25]. This fact enable us to search effects due to the residual charge-exchange interactions studying such transition. The shape factor of this $\beta$-decay has been measured several times in the last years [26-29]. Lipnik and Sunier [22] have considered the experimental data of Daniel and Schmitt [27] in their analysis. However the newer measurements performed by Hsue et al. [28] and Schneuwly et al. [29] do not agree with the shape factor presented in Ref. 27. They are consistent with the older experimental results published by Yamazaki et al. [26]. Although the authors of Ref. 29 have also included a brief theoretical discussion, their work remained essentially as experimental one. Therefore a detailed theoretical study of the second-forbidden $\beta$-decay of $^{137}\text{Cs}$ is very interesting. Mainly, the predictions of the CVC theory and the absolute values of phenomenological $\beta$-moments deserve a deeper analysis. The investigation about the absolute values of the n.m.e. might give information on the residual charge-exchange interactions.

The most important theoretical topics of second-forbidden $\beta$-transitions are summarized in Section 2. The method of analysis and the results are presented in Section 3. Finally, Section 4 is devoted to the discussion.

2. Theoretical Aspects

2.1. General Formulae

The expression for the spectrum shape factor $C_{\beta}(W)$ in the case of second-forbidden $\beta$-transitions is

$$C_{\beta}(W) = \frac{1}{\sqrt{5}} b_{22}^0 - \frac{1}{\sqrt{7}} b_{33}^0. \tag{1}$$

Here $b_{LL}^0$ are the particle parameters, which depend on the n.m.e. listed in Table 1, and on the electron wave functions. Following the paper of Hocquenghem and Berthier [30], the formulae for $b_{LL}^0$ can be written in this way,

$$b_{22}^0 = \eta^2 \sqrt{5} (x^2 s_1 + y^2 s_2 + u^2 s_3 + x y s_4 + x u s_5 + y u s_6), \tag{2}$$

$$b_{33}^0 = \eta^2 (-\sqrt{7}) z^2 s_7.$$

The quantities $s_1, \ldots, s_7$ are defined in terms of the usual combinations [31] of electron wave functions.