We consider a class of connected three-dimensional boundary-value problems for the quasi-linear heat equation that model the temperature fields in molds under a continuous cyclic molding of flat castings. We propose a method of approximate solution. The results of the computations are shown as the dependence of the dimensionless temperature on the time throughout ten cycles.

One of the fundamental problems that arise in designing automated production lines is the study of the thermal processes that occur in the continuous cyclic functioning of molds in a petrurgic casting - mold system. Problems of computing the forming elements of casting machines and the temperature fields are connected with the development of the effective methods of solving the boundary-value problems for a quasilinear heat equation in noncanonical domains. In attempting to find an exact analytic computation of the processes of hardening and cooling of a casting in a metal mold investigators encounter significant difficulties [2, 3]. Exact methods can be applied only for solving simplified boundary-value problems [1, 4].

In the course of modeling the cyclic processes at an arbitrary point of the domain in question repeated alternation of the stages of heating and cooling are observed, as a result of which the thermophysical coefficients of the material of the mold depend essentially on temperature and the boundary conditions depend on time. In this connection an effective method of computing a cyclic process can be based on the discretization of the boundary-value problem with respect to time, in order to determine more simply the stage of thermal work in which the mold happens to be and change the boundary values if necessary.

In the present article we construct a mathematical model of a cyclic thermal process in the form of boundary-value problems and also develop a method for numerical solution.

The original three-dimensional boundary-value problem in dimensionless variables consists of finding the solution of the quasilinear heat equation

$$\frac{l^2}{\beta(t)} \rho(u) \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(u) \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda(u) \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda(u) \frac{\partial u}{\partial z} \right)$$  \hspace{1cm} (1)

in a domain $\Omega$ with $t > 0$ under the initial conditions

$$u \Big|_{t=0} = u_0(x, y, z), \quad (x, y, z) \in \Omega$$  \hspace{1cm} (2)

(Fig. (1)) and boundary conditions (with $t > 0$)

$$\frac{\partial u}{\partial x} \bigg|_{x=0} = 0, \quad (y, z) \in \left\{ 0 < y < \frac{b}{l}; \ 0 < z < \frac{d}{l} \right\} \cup \left\{ \frac{b_0}{l} < y < \frac{b}{l}; \ \frac{d}{l} \leq z < \frac{d+H}{l} \right\};$$  \hspace{1cm} (3)

$$(a_1 l u + \lambda(u) \frac{\partial u}{\partial x}) \bigg|_{x=1} = 0, \quad (y, z) \in \left\{ 0 < y < \frac{b}{l}; \ 0 < z < \frac{d+H}{l} \right\};$$  \hspace{1cm} (4)

$$(a_2 l u - \lambda(u) \frac{\partial u}{\partial x}) \bigg|_{x=\frac{a}{l}} = F(t), \quad (y, z) \in \left\{ 0 < y < \frac{b_0}{l}; \ \frac{d}{l} \leq z < \frac{d+H}{l} \right\};$$  \hspace{1cm} (5)

$$\frac{\partial u}{\partial y} \bigg|_{y=0} = 0, \quad (x, z) \in \left\{ 0 < x < 1; \ 0 < z < \frac{d}{l} \right\} \cup \left\{ \frac{l_0}{l} < x < 1; \ \frac{d}{l} \leq z < \frac{d+H}{l} \right\};$$  \hspace{1cm} (6)

$$(a_1 l u + \lambda(u) \frac{\partial u}{\partial y}) \bigg|_{y=\frac{b}{l}} = 0, \quad (x, z) \in \left\{ 0 < x < 1, \ 0 < z < \frac{d+H}{l} \right\};$$  \hspace{1cm} (7)

Fig. 1

\[ (\alpha_2 u - \lambda(u) \frac{\partial u}{\partial y}) \bigg|_{y=\frac{b_0}{t}} = F(t), \quad (x,z) \in \left\{ 0 < x < \frac{l_0}{l}; \quad \frac{d}{l} < z < \frac{d+H}{l} \right\}; \]

(8)

\[ (\alpha_1 u - \lambda(u) \frac{\partial u}{\partial z}) \bigg|_{z=0} = 0, \quad (x,y) \in \left\{ 0 < x < 1, \quad 0 < y < \frac{b}{l} \right\}; \]

(9)

\[ (\alpha_1 u + \lambda(u) \frac{\partial u}{\partial z}) \bigg|_{z=\frac{b}{2}} = 0, \quad (x,y) \in \left\{ 0 < x \leq \frac{l_0}{l}, \quad \frac{b_0}{l} < y < \frac{b}{l} \right\} \cup \left\{ \frac{l_0}{l} < x < 1, \quad 0 < y < \frac{b}{l} \right\}; \]

(10)

\[ (\alpha_2 u + \lambda(u) \frac{\partial u}{\partial z}) \bigg|_{z=\frac{t}{t^*}} = F(t), \quad (x,y) \in \left\{ 0 < x < \frac{l_0}{l}, \quad 0 < y < \frac{b_0}{l} \right\}. \]

(11)

Here \( u \) is the dimensionless temperature: \( u = \frac{T - T_\infty}{T_0 - T_\infty} \), \( T \) is the Kelvin temperature of a given point of the field at a given instant of time, \( T_\infty \) is the initial temperature of the surrounding medium far from the body, \( T_\infty = 293 \text{ K} \), \( t = \frac{t}{t^*} \) is the dimensionless time, \( t^* \) is the characteristic value of the duration of a single casting cycle, \( t^* = 60 \text{ sec} \), \( x = \frac{\bar{x}}{l}, \ y = \frac{\bar{y}}{l}, \ z = \frac{\bar{z}}{l} \) are the dimensionless spatial coordinates, \( 2l \) is the maximum outside dimension of the mold, in meters, \( c \) is the specific heat capacity per unit mass in Joules per Kilogram-K, \( \lambda \) is the coefficient of heat conduction in watts per meter-K, \( \rho \) is the density in kilograms per cubic meter of the material of the mold, \( \alpha_1 \) and \( \alpha_2 \) are the coefficients of heat exchange with the surrounding medium and the melt respectively, in watts per square meter-K, and \( F(t) \) is the law of cooling of the surface of the ingot being formed, obtained from experimental data.

The original boundary-value problem determines completely the class of problems that describe all the stages of the thermal work of the mold. Thus when \( u_0 = 0 \), we obtain the problem of heating a cold mold during the first casting of a melt. When \( T_1(t) = 0 \) and \( \alpha_1 = \alpha_2 \), we obtain the problem of cooling the mold in a position without a melt. The heating of the hot mold in subsequent casting cycles is described by the boundary-value problem (1)–(11) in which \( u_0(x,y,z) \) is the solution of the preceding cooling problem. Thus these boundary conditions are interconnected, since the solution of each of them is the initial condition for the following one.

The difficulties that arise in solving such a class of boundary-value problems are determined by the following factors: 1) variation in the thermophysical characteristics as a result of oscillations of temperature over a wide range, which makes Eq. (1) nonlinear; 2) the complexity of the geometric shape of the domain.